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Edited by William David Reeve

The Contribution of Arithmetic to a Liberal Education

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Ginn and Company, Boston, Massachusetts

FIVE OR six persons devoted to education and hence possessed of more than average intelligence are having luncheon. Each has ordered a different meal; each is expecting to settle his own account. The waiter, however, disregarding the request for separate checks, has brought in but one bill and has placed it before the most prosperous-looking person in the group. You have doubtless participated in such a luncheon. If you have, you appreciate the comedy which ensues as these table companions try to break down the total into the proper amount for each person and assess the tip which each should pay. Everybody talks, nobody thinks, and nobody has the right change.

Consider this problem: " . . . radium is worth $25,000 a gram, with the present United States supply only 300 grams." What is the value of the supply in the United States? Is your answer $750,000, $75,000,000, or $7,500,000?

Educators who have worked with graduate students know how furiously they will figure and how little their figures mean to them. These students represent the best product of our school system, yet they are often surprisingly lacking in number sense. They can count, but they cannot think. The evidence is all too impressive that the teaching and the learning of arithmetic in the public schools have failed dismally to secure a widespread intelligence in the use of number.

On the other hand, you all know people who, although they may or may not have much formal schooling, possess to a high degree arithmetical competence. A definition of this type of competence is difficult, but it is easily recognized when seen in action. The person who possesses it has a lively sense of the meaning of number. He may not be able to say much about the theory of number or about place value in the Hindu-Arabic system or about the rounding off of numbers, but he has a working knowledge of these things, and he puts it to use. Observe him, for example, when in conversation an opportunity presents itself for multiplying 84 by 27. He hardly pauses in the discussion. He knows instantly the approximate order of magnitude of the product. He sees this as about 1,600 plus about 600, or something like 2,200. (Notice that in doing this, he has multiplied not from right to left as we teach children to do in school, but from left to right, as do most people who are really masters of number ideas.)

A man of arithmetical competence will note and use, without comment and without delay, the average size of 2,688,514 farms whose total area is given as

48,504,612 acres. He senses and reacts appropriately to the implied per cent that Germans, numbering 3.5 millions, are of a total population of 15 millions. Knowing that a meter is 39.37 inches, he never fails to place the decimal point correctly in case he needs the equivalent of a millimeter. If the vote in a given state for his presidential candidate is reported as 564,709 against 275,418 for the next candidate, he instantly enriches the statement by the thought, "Beat him by 300,000," or "Licked him two to one."

Moreover, the person who is literate with respect to arithmetic possesses a considerable amount of numerical tradition or lore. The items which make up this lore should be matters of common knowledge because such knowledge adds to one's understanding of the world and the people in it. A pound sterling is about \$5, and a French franc is about 3¢. Letter paper is $8\frac{1}{2}$ by 11 inches. Filing cards come in the sizes 3×5, 4×6, and 5×8 inches. Standard sizes of rugs are 6×9 feet, and 9×12 feet. An acre is about 44,000 square feet. The average height of Anglo-Saxon men is 68 inches. Tables and desks are 30 inches high, and chairs are 18 inches. Under standard conditions, sound travels 1,190 feet a second, and light travels 186,000 miles a second. Eighteen miles to the gallon of gasoline is "good going." A property tax rate of \$30 a thousand on full valuation is high. Six per cent on new loans is ordinarily not expected today. A driving speed of more than 50 miles an hour is dangerous.

Life in modern society is full of norms and standards. We are accustomed to suppose that measures of type and variation, such as averages and standard deviations, are only to be had through statistical manipulation. As a matter of fact, the experience of modern life provides us all with quantitative ideas of type and variation. Unconsciously our experience puts meaning into the phenomena about us in much the same way that the statistician puts meaning into his observations through conscious manipulation of their measures.

And this isn't all. In ordinary social intercourse we have need of number. We can engage in no serious conversation without the necessity for understanding numerical relations. Getting the idea depends upon following the thought of the speaker, and you can't follow if you don't know where you are numerically. You don't even know when to agree, when to keep still, when to protest, or when to say "oh, my!" You may not even be in the right decimal place. You may be thinking in thousands when you ought to be thinking in hundreds. And this brings me to a topic which I think is exceedingly important in this matter of liberalizing arithmetic.

One of the great words in arithmetic is *approximation*. I was lately shown a portion of the key to an arithmetic in use some sixty years ago. The answers were given to the ninth decimal place. Would anybody contend that accuracy to billionths had any value except as a way of getting practice in computation? In life most numerical results are not exact. Measuring is always subject to error, so much so that a by-no-means unimportant topic in arithmetic for a liberal education should deal with the size, importance, and significance of error. Even counting may be wrong. No one, for example, supposes that the 1930 population of Boston was exactly 781,188, as reported. Therefore, in our handling of numbers, we should seek the degree of accuracy which will serve our purpose. Anything beyond that is a waste of effort.

There is another reason for intelligent approximation. When you hear an argument or read an exposition involving quantitative statements, you cannot follow the speaker or writer unless you can *interpret* his numbers. In order to do this, you must apprehend the numbers quickly and easily. It is hopeless, however, and it is unnecessary, to attend to all of the digits. In the case of large whole numbers you will find it sufficient to deal with a few

of the left-hand symbols. The rest you will disregard, and this is the same as saying you will treat them as zeros. In the case of decimals, you will attend to one or two significant figures only; and again, these will be at the left.

Then how many of the left-hand digits should you regard: one, two, three? There is no rule. The size of the number, the connection in which it occurs, the purpose that it is to serve, the reliability of the source from which it was derived—all these considerations play a part in deciding how, as we say, to round off the number. Here, then, is a place for the practical numerical judgment which the arithmetically literate person knows how to exercise.

This kind of judgment does not arise through drill in computing. It can only be had through the practice of judgment under defined conditions. One good way to develop this sort of judgment among pupils is to provide for floor talks and committee reports in which the object is to treat quantitative data effectively, and in which the hearers seek to interpret the same data. Another way is to provide reading matter which not only involves numbers but which also requires the combining and comparing of numbers.

The point I am making is that in following a speaker or in reading printed matter we approximate by using a few left-hand figures. If we are skilful, we exercise critical judgment as we do this. Moreover if we compute as we go along, we do so, not from the right as we were taught in school, but from the left. We do so for two reasons: First, we are thus using the digits of highest value, and second, the direction of our reading is from left to right—that is, we arrive at the left-hand figures first.

Alas, however, this ability is not common. A good deal of experience is needed—experience of a kind seldom furnished in an arithmetic class—before one can read or hear quantitative data with ease and enjoyment. There is no surer way to kill a speech than to put figures in it. This is not because the figures are unimportant. They may well be the most important part of the message. It is because so large a part of the audience lacks that part of a liberal education which arithmetic, rightly learned, can confer.

The task of the school is too exclusively understood to be that manipulation of symbols which we call computing. The school's task, in respect to arithmetic, is in reality far more fundamental. It is nothing less than an attack upon arithmetical illiteracy. There is an illiteracy applicable to quantitative ideas just as there is an illiteracy applicable to the generalizations and concepts expressed in words instead of in figures. In each case competence, or literacy, is something more than the manipulation of the symbols. It is an appreciation of the meaning attached to the symbols and an ability to apply the symbols in order to facilitate thought.

I like to think that the restless gropings of educational thinkers during the past twenty-five years have finally emerged into a conception of arithmetic which is no longer satisfied with the speed-and-accuracy formula of the second and third decades of the present century. Speed and accuracy, it appears, are partly by-products—by-products of an ability to see and appreciate the number in life about us, an ability to think in quantitative terms, and an ability to act intelligently under the guidance of number. The lack of this ability I am calling arithmetical illiteracy. The possession of this ability I regard as a unique item in a liberal education. What I am therefore saying is that arithmetic, as it is rapidly coming to be regarded, is in a large and, indeed, in a thrilling way a contribution to a liberal education.

This conception of arithmetic is making over the whole subject. Moreover it is giving it an importance it never possessed before. Arithmetic is no longer a bag of tricks; no longer something to puzzle little children. It takes on something of the dignity that arithmetic had as one of the

seven liberal arts in the curriculum of ancient Greece. It does this while, at the same time, it is quite different from, and superior to, the subject matter to which the Greeks applied the term "arithmetic."

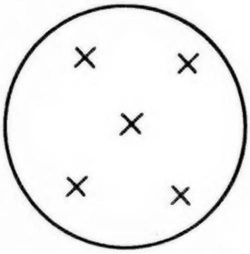
The arithmetic which I regard as contributory to a liberal education begins early in the course of study and lasts a long time. It is a proper field of teaching and learning for the first-grade child, and it might well receive greater attention than is now being devoted to it in the junior and senior high schools. In my judgment, those who have participated in what I have called a flight from arithmetic have done so because they have conceived of arithmetic in narrow, not to say mean and unworthy terms. If the task of arithmetic is merely to develop computers in the sense of manipulators of symbols, where rule of thumb is the only guide and where drill is the only method, then a flight from arithmetic is justified and those who say, in substance, "This is a dreary subject and a difficult one. Let's postpone it until the third, the fifth, or the seventh grade," are quite logical in their conclusions. The trouble is not with their logic but with their premises. Their assumptions are wrong. Change these assumptions, give to arithmetic the dignity and importance not only of a major contributor but of a unique contributor to a liberal education—do this, and the flight from arithmetic is no longer possible. Thought and action take an entirely different direction. The sterile discussions which have characterized the literature of arithmetic so long, mere changes of method to an unchanging end, become trivial. Larger purposes bring more penetrating analysis of means to higher ends. We are driven back to a deeper consideration of our postulates.

For example, what is number? In the accompanying diagram of a group of crosses, where is 5?

Is it here? Is it there? Is it in between the objects? Is it all over the area within the bounding line? If you rub out one of the crosses and put another, perhaps of a different color, in its place, is 5 still there? If you rearrange the crosses so that they form a different pattern, is the number altered?

Evidently this mysterious thing we call number is, when it is applied to objects, unaffected by substitution or rearrangement.

How do we apprehend this number? If I ask you, "How many crosses are here?" you unhesitatingly say, "Five." Most first-grade children will be equally successful in answering this question.



In seeking to answer the question, "How do we apprehend this number?" I am aware that I run the risk of loading my paper with a long and forbidding statement. It will serve my purpose, however, and will save a strain upon your attention, if I show quite briefly that what is going on when one apprehends five objects in such a diagram is one of the most wonderful operations in nature. Moreover, it is not one operation: it is two. Further, these two operations are mutually antagonistic. The mind must give attention to all the objects taken together. But this is not enough. It must also give attention to the individual objects; and apparently these two emphases must take place together. Thus we have the generalization which leads to contemplation of the whole group and the simultaneous exclusion of anything outside the field under consideration (any cross, for example, or other object which might lie outside the bounding line); and besides this generalization we have an analysis, the very opposite of generalization, in virtue of which each object is taken into consideration. There can be no apprehension of the number of items in a

group without this double and antagonistic functioning of synthesis and analysis.

"But what of it?" you may ask. Nothing, perhaps, unless we go further. Let us, therefore, take the next step.

It is evident from what I have said that number is man-made. In connection with the diagram I have raised the question, "Where is 5?" The proper answer to this question is that 5 is in the mind of the beholder. Although number is used everywhere in the world, it has no material existence anywhere. It is only as man in pursuing his purposes has manipulated things having extension and plurality that he has developed number ideas. As Wheat says, "Objects in the world do not make themselves countable; the child counts the objects which surround him only when he has learned to count. Playing store doesn't project addition to be beheld and studied; the child keeps accounts only when he has learned to add. Business practices do not of themselves portray percentage; the idea of percentage is brought to business practices to make them meaningful."

Now if number is not found in nature, but is man-made, if it has risen in the mind of man as he has manipulated things to serve his needs, we have a fundamentally different notion of the way to proceed in school than we have when the prime object of arithmetic is held to be the manipulation of symbols with speed and accuracy. As matters stand now, it is clear that the essential thing is to give the child experience out of which he may build number ideas for himself; for nothing is surer than that number ideas, to be vivid, flexible and versatile, must arise from first-hand experience. In their most useful form, these number ideas must be highly abstract and quite devoid of concrete loading; but such ideas develop slowly, growing with the growth of the child. Abstractions, such as "6," or "8+5=13," cannot be handed to the child by someone else. The child must achieve them himself.

Obviously, this takes time. Therefore if arithmetic is to serve its best purpose, it must begin early. I have no patience with those who would defer the right kind of arithmetic to the middle grades. Concrete arithmetic should begin as early as any educative experience begins.

There is another implication in the nature of number and in the method by which control of it is acquired. The child's experience at every point should be characterized by natural wholeness rather than by artificial fragmentation. The accepted psychology of arithmetic has lately undergone complete transformation. Ten years ago leaders in the field of arithmetic devoted much effort to the minute analysis of subject matter into parts. They called these parts skills. There were big skills and little skills, the latter being parts of the former. Mastery of a given topic was held to be the total of subordinate masteries within the field.

Most competent students of arithmetic now look differently upon the learning of the subject. They give first consideration to wholes and only subsequently to parts because they believe that wholes give meaning to parts. For this reason teaching takes on a different character. The earliest teaching of a given topic is characterized by wholeness. I do not mean that this wholeness is all-inclusive, but rather that it is wholeness in certain particulars. I recognize that a topic as offered at any given time may be a whole in certain respects, whereas in other respects it may be partial or incomplete.

Consider, for example, the treatment of fractions. Even at an early level—and there is no reason why we may not begin to treat fractions as early as Grade One—our presentation of the subject may be well rounded. The commonest, easiest, and most useful series of fractions is that which arises from halving, or, as early mathematicians used to call it, mediation. Halves, fourths, and eighths are almost as common as the smallest whole numbers. If our early teaching of fractions is limited to this series, our treatment in all other respects may have a surprising degree of

completeness. If desired, all four operations may be exemplified—of course, concretely. Similarly the equivalents among fractions of this series may be handled. The idea may be concretely illustrated, even if not put into verbal form, that if you multiply one term of a fraction, you must multiply the other by the same number.

The young child has by him a capital instrument by means of which his experience in the handling of this first series of fractions may be enriched. He has a ruler. Normally it is divided into halves and quarters, or into halves, quarters, and eighths. On his ruler he may add, for example, $\frac{1}{2}$ and $\frac{3}{4}$. He may subtract $2\frac{5}{8}$ from $3\frac{1}{4}$. Notice if you please, that in each of the cases I have mentioned he is using unlike fractions. He doesn't think of them as unlike. He is using an all-around situation in which fractions of the same denominator and fractions of different denominators may occur in their proper setting without being considered as demanding separate "skills."

Again, by means of this instrument the child may do concrete multiplication. He may take $\frac{1}{2}$ three times and read off the result from his ruler. He may multiply $1\frac{1}{4}$ by 2 and actually see the answer He may even use a fraction as a multiplier, finding, for example, $\frac{1}{2}$ of $1\frac{1}{2}$. He may do this without any sense of difficulty, and of course he doesn't know that he is multiplying. We need not hasten to teach him our terminology although I, for my part, do not deprecate the use of technical terms early in the pupil's school career, provided the meaning of them is understood.

Finally, the ruler permits the child to divide. On it he may obtain the answer to such a question as, "In 2 there are how many fourths?" First he finds 2 on his ruler; then he counts the number of $\frac{1}{4}$ inches required to make up 2.

I have used the ruler merely as an illustration. There are many other ways that can be used, and should be used, to give the child early experience with fractions. I am not so much interested in exhibiting the richness with which this type of work may be done as I am in showing first, that according to modern conceptions of the learning of arithmetic the work should begin early; and, secondly, that it should be characterized from the beginning and throughout by the idea of wholeness rather than of fragmentation.

There are so many things that might be said, as soon as one approaches arithmetic with the idea of making it a liberalizing discipline, that one scarcely knows where to begin or end. I should not like, however, to close this discussion without urging upon you the fact that arithmetic so viewed is essentially a social study. It is a social study because it is the only area of learning available to all the people for the establishing of order, of system, of accuracy, and of punctuality. Moreover, amid all the diversity of languages, the one symbolism common to civilized men everywhere is the symbolism of arithmetic.

I hardly need to tell you that number is explicit and implicit in all the affairs of life. A familiar aphorism has been that whatever exists, exists in some amount. We live, therefore, as Hogben says, "in a welter of figures; cookery recipes, unemployment aggregates, taxes, war debts, over-time schedules, speed limitations, bowling averages, betting odds, calories, babies' weights, clinical temperatures, rainfall, motor records, power indices, gas meter readings, bank rates, freight rates, death rates, discounts, interest, lotteries, and wave lengths. . . . Ratios, limits, and acceleration are not remote abstractions dimly apprehended by the solitary genius. They are photographed on every page of our existence."

As we pass in these days from an economic era of *laissez-faire* to one of scientific and, to a certain extent, of political control, it is clear that our citizenry must, as never before, acquire a new sense of mathematical power, insight, and confidence. Without this power, insight, and confidence we may easily fall victims to a certain statistical maladroitness—or is it

propaganda?—which greets us in print and on the air. News item: The Garand rifle for the army is now being turned out at the amazing rate of 1,000 a day. We are expected to take satisfaction from this fact. Our satisfaction, however, ceases if we compare the fact with the job to be done. At the rate of 1,000 rifles a day, it would take 1,000 days, or nearly three years, to equip a million men. It would take $5\frac{1}{2}$ years to make two million rifles—a practically necessary number for the expanded army, if reserves for replacement are to be provided. Are 1,000 rifles a day enough?

Again, the numerically illiterate may be mystified by the announcement of per cents of increase in the production of munitions. Not long ago the OPM gave out that our production of light tanks had increased 1,260 per cent during the second quarter of 1941 over the first quarter. We were supposed to say "Oh, my" to that. But as a matter of fact the figure means nothing unless we know something about the production in units for the first quarter. That production was known to be low because the army was turning to a new model. If only 5 tanks were turned out in the first quarter and 70 in the second, the difference, 65, would be an increase of 1,300 per cent—but it wouldn't be many tanks and it wouldn't be anything to cheer about.

Too often in the past it has been said, "Let the experts do our figuring," or "Let the calculating machines add and subtract for us." The world is full of tragic failure because folks let some one else do their figuring. The sharper and the mountebank, the swindler and the confidence man are good at figures.

Arithmetic is a social study for another reason. It has a remarkable history. The evolution of our number system is an illuminating chapter in social progress. We accept this number system, as we accept the air about us, without bestowing upon it the admiration which it deserves. For thousands of years within the period of recorded history—and for no one knows how long before—man was obliged to do his computing by means of an instrument. He used stones, knotted strings, sticks with notches in them, and finally a form of abacus; but nowhere, until the Hindu-Arabic system with its zero as a place holder came into use, did anyone have a notion of performing operations with the number symbols themselves.

Thus it came about that our form of notation actually expressed number in a way to assist operational thinking. There is no such assistance to thought in the manner of expressing Roman notation, nor was there any such assistance when, as in Greece, every letter of the alphabet represented a number.

Moreover this notation has been a powerful contributor to the development of mathematics. Not long after the Hindu-Arabic system became generally known in Europe, that remarkable development of the system to the right of units' place, which we call decimals, was invented. Then came a further development still due to our peculiar place system, namely, the development of logarithms, the idea that any number may be represented as a power of 10. Without these direct developments of our system of notation, mechanics and astronomy would be crippled; physics, navigation, and surveying would not advance.

Again, our system of notation has contributed enormously to the management of modern affairs. It is difficult to see how large enterprises with their complicated systems of records and accounts could be carried on without the brevity and portability of numerical data which our notation permits. The heavy demands of modern business upon computation of all kinds are generally recognized. The clumsy systems which preceded the Hindu-Arabic system would be utterly incapable of meeting these demands. Much of our scientific work, depending as it does upon the massing of numerically

expressed observations, would likewise be handicapped.

If our system of notation is as important as all this, it is clear that our curriculum in arithmetic should take due account of this fact. I venture to say that we are not doing so. Not only a new chapter in arithmetic, but a new point of view throughout the entire range of the subject, would be justified by the significance of the number system as a system.

It is equally clear that we do not spend nearly enough time and effort—or rather that we do not see that our pupils spend nearly enough time and effort—in dealing with number concepts.

Children may know numbers as groups of objects arising from counting, as aggregates derived from smaller numbers, or as ratios. Functionally they may know their numbers so that they can reproduce them in action or, on the other hand, identify them when the conditions to which they correspond are presented.

If a child knows the number system and if he has well-grounded number concepts, he will never have to learn as an abstract rule the fact that three-fourths of a number is three times as much as one-fourth of it, nor the fact that dividing by one-third is numerically the same as multiplying by three, nor the further fact that dividing by two-thirds will be half as much as dividing by one-third. He will not misplace the decimal point in dividing because his lively sense of the meaning of numbers will make any but the correct location of the point an absurdity. He will be able to estimate in advance the approximate size of his results, and he will be furnished with a critical instrument to apply to his results after he has obtained them.

Closely related to the thought that the number system and number concepts are proper objects of teaching in a liberalized course of arithmetic is the further thought of the contribution which arithmetic can make in a broader sense through the fact that it is a system rather than a series of unrelated items. All its parts hang together. In order, therefore, to obtain this liberalizing result, arithmetic must be learned as a system. We have it from Dewey that "training by isolated exercises leaves no deposit, leads nowhere; and even the technical skill acquired has little radiating power or transferable value."[1]

These are magic words: radiating power, transferable value. When we teach arithmetic with such power and such value, we have what I am calling a contribution to a liberal education.

[1] John Dewey, *How We Think*, p. 191. Boston, D. C. Heath, 1910.

The Annual Meeting of the National Council!

THE annual meeting of The National Council of Teachers of Mathematics will be held, as planned, in San Francisco, California on February 20–21, 1942 at The Palace Hotel. See pages 41–44 of the January (1942) issue for the program.

Mathematical Tables!

THOSE who desire to receive announcements concerning new mathematical tables as they become available should write to the Information Section, National Bureau of Standards, Washington, D. C. asking that their names be added to the mailing list maintained for this purpose.

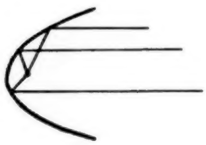
A Lesson on the Parabola,* with Emphasis on Its Importance in Modern Life

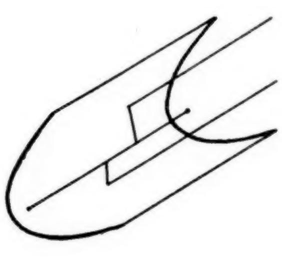
By Chester C. Camp
University of Nebraska, Lincoln, Nebraska

While vacationing in the North Woods of Minnesota last summer I received a letter from your President asking whether I would give a talk before the Mathematics Section of the Nebraska State Teachers Association in October on "The Parabola." Of course I accepted but when I returned to Lincoln in September I found opposite my name the title, "*A Lesson* on the Parabola." The more I thought about it the more I wondered just who it was that this *Lesson* referred to, whether to the speaker, or to your President, or to the audience—you know if a person learns his lesson he usually doesn't make the same mistake again!

If any of you are interested in antiques you certainly should add the parabola[1] because it goes away back to about 340 B.C. when it was discovered by Menaechmus. But he used a different kind of cone, one with a right angle at the vertex, and the curve wasn't called a parabola. It was known as the "section of a right angled cone." But just because the parabola is an antique, do not get the idea that it is antiquated. On the contrary it is used more today than any other curve except the circle, and it is the simplest curve to treat apart from the circle. I mention only two of its many applications now but will mention others later.

The first is the headlight of an automobile. It has parabolic sections (see sketch), such that if the bulb is placed at a certain point every ray of light which hits the parabolic reflector is bent so as to go straight forward in the same direction. I shall explain later why this is true. The second is a reflector in the form of a parabolic cylinder (see second sketch), which is the main part of the Solar Engine, six feet long and two feet wide, one of six types invented by Dr. Charles G. Abbott, Secretary of the Smithsonian Institution. Here the scheme is reversed. A ray of light





coming in from the sun strikes the parabolic surface and is reflected to some point along a tube containing a black liquid, aroclor, which absorbs the heat. This liquid is pumped out and the heat produces a steam engine of $\frac{1}{2}$ H.P. Mr. Abbott claims that fifteen per cent of the heat striking the reflector is turned into useful work. His idea is ultimately to utilize the heat of the sun commercially (especially when our coal supply is exhausted). Over half a million dollars ($647,000 in fact), have been given to the Massachusetts Institute of Technology to work on this idea. One of his other inventions is a solar oven which bakes gingerbread and automatically indicates when it is done.

* Based on an address delivered by invitation before the Nebraska State Teachers Association at Lincoln, Nebraska, October 27, 1939.

[1] Here the speaker took a wooden model of a cone apart to display the curve.

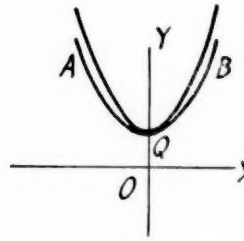
When a High School teacher takes up the parabola in connection with the quadratic it furnishes a wonderful opportunity to interest all students in the simple and important examples of parabolas which surround them and the more gifted student in higher branches of mathematics such as analytical geometry and the calculus. These are pointed out as the fields in which we prove that the curve is a parabola, derive its properties, find the area of a segment and the length of the curve between two points. One may also rotate the curve and find the volume of a segment cut off by a plane as well as the area of the surface of this solid.

If one takes the simplest case $y=x^2$ it is not difficult to get a student to plot points for $x=0$, 1, 2, 3 and their negatives and if one makes him plot enough points including fractional values one may prevail on him to draw a smooth curve through the points instead of connecting them with straight lines. If he asks why one draws a smooth curve you can tell him that is one of the things we prove by the calculus in connection with continuous functions. One sees from the symmetry that the y-axis is a line of symmetry. It is called the axis of the parabola and the lowest point is called the vertex. If one sketches the curve $y=ax^2$ where a is positive one obtains a similar curve for, since $y/a=x^2$, obviously this is equivalent to changing the scale on the y-axis. If a is negative then the curve opens downward and has its vertex at the top.

A student may ask you[2] whether holding a chain by its two ends will give you a parabola. You can tell him that this can be determined by the use of calculus. Upon being questioned in regard to this after the speech the speaker explained that the equation for the chain derived by the calculus is

$$y=a(1+x^2/2!+x^4/4!+x^6/6!+\cdots)$$

The right hand member is an infinite series and the equation represents a curve called the catenary which opens upward if a is positive. Although it resembles the parabola it may be shown to be a different type of curve as follows:



Consider the parabola $y=a(1+x^2/2)$, represented in the figure by AQB, whose right member consists of the first two terms of the infinite series of the catenary. It has the same vertex and may be shown to have the same radius of curvature at the vertex. However the graph of the catenary lies above and inside this parabola except at the vertex. This is true because all the remaining terms of the infinite series are positive, which give a larger value for the y of the catenary.

Euclid who wrote about 290 B.C. didn't add much to the theory of the parabola. He continued to call it "the section of a right angled cone" and so did Archimedes (circa 250 B.C.). The latter did a very remarkable thing, however. By using his so-called method of exhaustion and the theory of moments he was enabled to find the area of a segment of the curve cut off by a straight line. This is usually done by integral calculus, which was not invented until the seventeenth century. He also furnished the first case of an infinite series which has come down to us, namely $\frac{1}{4}+\frac{1}{4^2}+\frac{1}{4^3}+\cdots$. He used it in connection with the area of the segment of "the section of a right angled cone" and found its sum to be one third.

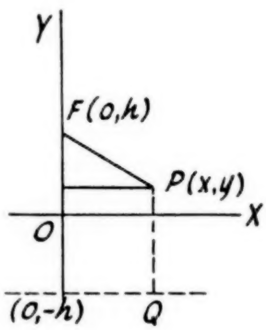
It remained for Apollonius (c. 230 B.C.) to find out that the parabola can be obtained by cutting any circular cone at the proper angle, i.e. by a plane parallel to an element of the cone[3] also that the ellipse

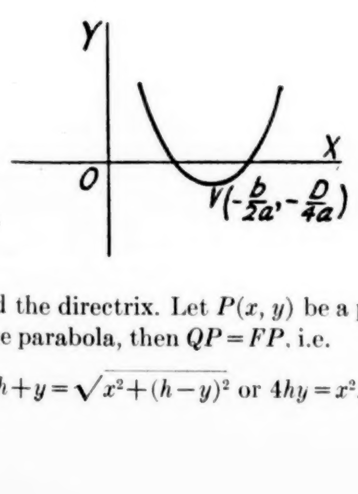
[2] At this point the speaker held a watch chain by its two ends.

[3] At this point the speaker showed the wooden model again.

and hyperbola may be obtained by orienting the cutting plane. It was necessary then for him to find new names for these curves. His names were the Greek forms of our present English words for the curves. So after over a hundred years the parabola was given a Greek name equivalent to the word "parabola." Apollonius wrote eight books on Conic Sections, four of which have come down to us and three more are available in Arabic. The first four are the more important, however.

It was not until about 300 A.D. that Pappus gave the definition which is now used in analytical geometry to define the parabola. It is given in terms of a point moving in a plane. We know that in space a fixed point and a fixed straight line not passing through it determine a plane. A point moving in this plane so as to be always equidistant from the fixed point and the line is a parabola. We may illustrate this by taking $F(O, h)$ for the fixed point called the focus, and a horizontal line h units below the x-axis for the fixed line,





called the directrix. Let $P(x, y)$ be a point on the parabola, then $QP = FP$. i.e.

$$h+y=\sqrt{x^2+(h-y)^2} \text{ or } 4hy=x^2.$$

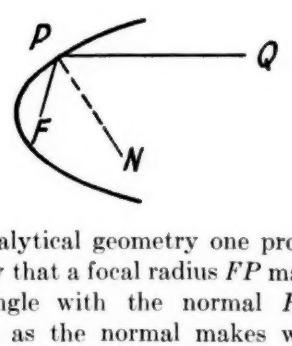
The more general form $y=ax^2+bx+c$ may be written for convenience as

$$y=a\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a^2}\right].$$

If a is positive the smallest value of y occurs when $x+b/2a$ has the value zero since the square of this quantity is always positive or zero. Hence $x=-b/2a$ for the minimum point and $y=-D/4a$ where $D=b^2-4ac$ is the discriminant of the quadratic equation $x^2+bx+c=0$ (see sketch). If a is negative a similar argument will show that these values of x and y will locate the vertex, which will be a maximum point on the parabola opening downward.

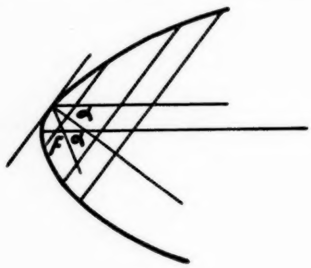
There are many problems of importance in which one needs to find the minimum of a function. One such is a cost function. Another application is that in connection with least squares in statistics. By the method just given one finds these minimum values algebraically. One may also find these points quite easily by calculus.

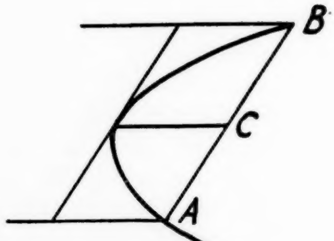
If one varies c one merely raises or lowers the parabola as a whole without changing its shape. The effect of changing b is to move the vertex horizontally as well as vertically. If $D>0$ the curve cuts the x-axis in two points. If $D=0$ it touches it and if $D<0$ there are no real crossing points. It is clear that the roots of the quadratic equation may be read off the graph approximately if they are real.



In analytical geometry one proves the property that a focal radius FP makes the same angle with the normal PN (see sketch), as the normal makes with the horizontal line PQ (a diameter of the parabola), which cuts the curve at the same point P. Since a reflected ray is always bent so as to make equal angles with

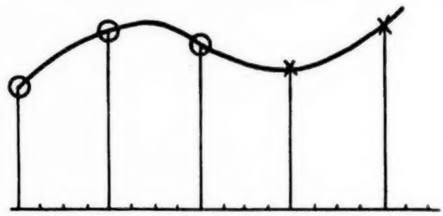
the normal the parabolic reflector is explained. The same property will enable us to find the focus of a parabola when the curve alone is given. Cut the curve by three parallel chords (see illustration). The midpoints of these will determine a straight line parallel to the axis of the parabola.





Another parallel drawn where this diameter cuts the curve will be tangent to the curve. Drawing the normal, one constructs an angle on the opposite side of the normal equal to α, the angle between the normal and the diameter. The line so determined will pass through the focus. Next construct chords perpendicular to the diameter. Their midpoints will determine the axis of the curve, upon which the focus must lie.

Consider a parabola cut by an oblique chord AB, at both extremities of which diameters are drawn as shown in the last sketch. Draw a third diameter bisecting the chord at C and through the point where it cuts the curve draw a line parallel to the chord. The segment of the parabola has an area equal to two thirds of the circumscribed parallelogram. I wish to make two applications of this. If any of your students are interested in farm problems they may notice that the end of a hay stack sometimes looks like a parabola. In such a case one may find approximately the volume of hay in the stack by taking two thirds of the product of the width at the bottom, the height and the length. The second is Simpson's rule which enables one to find approximately the area under any



curve (see sketch) by cutting it vertically by straight lines equally spaced and sufficiently close together. One needs an even number of strips. Pass a parabola through the tops of the first three ordinates. This can be done by substituting the values of x, y for each point in the equation $y=ax^2+bx+c$ and solving the resulting three equations for the three unknowns a, b, c. Pass another parabola through the tops of the third ordinate and the next two. By continuing this process one may find the area sought by taking the areas under the parabolas. The combination of all is a comparatively simple rule known as Simpson's rule. Any desired accuracy may be obtained by taking the distance between successive ordinates sufficiently small.

There are many other applications of the parabola including paths of projectiles which may be shown by calculus to be parabolas if friction of the air be neglected, parabolic arches, suspension bridges in case the horizontal distribution of the load is uniform, and certain phases of acoustics. The Hill Memorial Auditorium at the University of Michigan is built so a speaker standing at the focus of a paraboloid of revolution whose upper part forms the end of the building behind him has an extreme advantage. All the sound waves going back are reflected straight out into the audience. This is not so much a boon to an orchestra since it is not so easy for them to

be concentrated at one point, unfortunately.

In teaching mathematics as in the case of other subjects there are many things we may do. If we can give a student an appreciation of art whether he can produce art, an appreciation of music whether he can compose it, an appreciation of literature whether or not he can create it, we have enriched his life. With mathematics we have the same opportunity. If we can give him an appreciation of the subject matter of mathematics, we have made his life richer, even though he may not be able to discover or invent new things in mathematics.

By connecting the quadratic function, which seems so abstract at first, with simple but important things that surround him in life[4] we may be able to change his attitude towards algebra to one of appreciation. If he is a student who has ability in mathematics we may be able to interest him in more advanced mathematics at this time, able to give him an interest that he will carry forward into his college life. If he is one of the eighty per cent who feel very certain while studying High School mathematics that they will never go on into a school that teaches higher mathematics, he may be one who can be greatly helped. If the teacher can make the very cold abstract equation of the parabola take on life and energy, it may become a very dramatic thing for him. If when he sees a quadratic and is asked to work with it he can see in it the features of a headlight or solar engine, visualize the cross section of a hay stack or auditorium, or imagine the path of a projectile or the cable of a suspension bridge, the teacher has been successful. Or more important still, if the student can think of it the other way round, that is, if when he sees any of the illustrations mentioned above he is reminded of the quadratic equation and it becomes a vivid thing to him, not only has his life been enriched, but we have given to this individual an additional appreciation of life in general. A similar thing can be done for the straight line and other mathematical formulae as well as for the parabola.

I have limited this discussion to the second degree parabola because that is the one we think of when we say *the parabola*. However during the past 2000 years since the Greeks introduced us to the very simple form other kinds of parabolas have been discovered. There are cubical parabolas and parabolas of the fourth degree. These are used in statistics to fit other curves. In fact there are parabolas of any degree n where n is an integer. There are even parabolas of fractional degree, for instance the semi-cubical parabola $y=x^{3/2}$, which has important properties. And I suppose that 2000 years from now when the Nebraska State Teachers Association meets in Lincoln there will be still more general parabolas with much higher properties!

[4] For motivation of other phases of mathematics see the author's article, "Contributions of Mathematics to Modern Life," THE MATHEMATICS TEACHER, vol. XXI, No. 4, April, 1928, pp. 219–226.

Defense

They may call math prosaic
And prate of other isms,
But have you seen the rainbow
That dances in my prisms?

They say a straight chalk line
Has lost its usefulness;
But crooked paths have never
Traveled towards success.

They say that feet and inches
Should not stray far from school
But did you know in Heaven
They use the Golden Rule?

They may scorn the circle
And say its use is dead,
But have you seen the halo
Above our Saviour's head?

From Shaker High School Verse; An Anthology by Students of Shaker High School

Reprinted by permission from the January, 1936 issue of PROGRESSIVE EDUCATION

Geometry for Everyone

By Kenneth S. Davis
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There are many geometric relationships and figures in our world that are a part of the experiences of every person, but they escape notice as such because they have not been brought to the attention of the observers. Or should it be said, those who do not observe.

Much has been written on the general subject of the functional aspect of our curricula. Many men in the general field of secondary education advocate the inclusion in the various curricula of those experiences which will contribute to the immediate enlightenment of the pupil. They question any material which in their opinion does not so contribute. Geometry has had its share of the questioning.

The writers in the field of mathematics education have also urged upon the teachers of mathematics the necessity of supplying more functional material to their pupils. Mathematics that is useful, mathematics related to the experiences of the pupils has been their text and sermon. However, mathematics teachers, trained by subject matter mathematicians, have been slow to respond. The result is that too many mathematics teachers give their pupils the material presented in the textbook and little else.

Even if we could not see the logic of these suggestions as mentioned above, we should be compelled to give notice to them because of the agreement in their content. The administrators, curriculum specialists, and mathematics supervisors are all in harmony in their desire for material that is functional. They want material that has relationship to the environment of the pupils. If we turn our attention to geometry, we see many parts of our environment that have geometric relationship. These will give us an opportunity to relate some parts of the pupil's experiences to this study of geometry.

The following is a partial list of some of these environmental parts that can be used for this purpose. This list is followed by some geometric statements which are keyed to the list. This is done so that readers may see the theoretical geometry that can be related to the parts of environment named.

1. Cylindrical cans for canned goods—for a given volume it will take less material for a cylindrical can than for any other type can. This can be shown by application of the volume and surface area principles developed in solid geometry for both cylindrical and rectangular cans. A given length of line will enclose more area when it is in the shape of a circle than when it is square or rectangular in shape.
2. Diagonal braces for gates—these form triangles in the gates. Triangles are rigid figures because they are absolutely determined by three sides Four sided figures may change shape unless braced.
3. The carpenter's plane—when a board is smooth the plane will cut all along the surface. The definition of a geometric plane is stated in that if any two points in a plane are joined by a straight line, the line will lie wholly in the plane. Thus the plane will cut all along the surface only when the surface is a geometric plane.
4. The carpenter's square—a right angle is formed when two lines meet perpendicularly. The square gives us square corners or right angled corners.
5. The arrangements of the contents of a room for the best effect or balance, thus giving function to the geometric principle of symmetry.

6. Physical symmetry of human beings—if a line is drawn from the center of the forehead to a point midway between the feet, an almost exact symmetry is found, two arms, legs, eyes (one on each side), one nose, mouth, chin, neck, body (balanced on this line)
7. Kodak pictures—similar figures have the same shape but differ in size. All dimensions are proportional.
8. The gable end of a roof—most gables of buildings illustrate an isosceles triangle, that is, two of the sides are equal. There are some exceptions to this, however, especially in houses built on the story and a half type of architecture in that windows are inserted in one side of the roof. Even in these, in many cases, a relief line is brought down to give the symmetrical appearance.
9. Rows of shingles on a roof—the edges of the shingles are usually parallel to each other as an aid in construction and to improve the appearance.
10. Wheels—wheels are circles and usually have circles within circles all with the same center. These are called concentric circles.
11. Plasterer's straightedge—again the definition of a plane is apparent here A line joining any two points in a plane lies wholly in the plane. The plasterer passes his straightedge over the wall in all directions, agreeing with the word 'any' in the definition, in order to insure a smooth wall.
12. Reflections in mirrors—the matters of angles of incidence and reflection, ratio and proportion, and plane symmetrical solid figures are the geometric principles involved in the act of looking at ourselves in mirrors.
13. Tree leaves—nature illustrates her understanding of line symmetry in the leaves of the trees. Blight and weather conditions sometimes affect this state of balance.
14. Sprocket wheels and chain on a bicycle—the wheels are circles with the chain forming common external tangents.
15. A sheet of paper—most pieces of paper are rectangular in shape and can be used to illustrate various properties pertaining to rectangles.
16. Two gear wheels in mesh—the gear wheels of an egg beater or the gear wheels in practically all pieces of machinery are illustrations of two externally tangent circles.
17. A baseball diamond—the field upon which baseball is played is a square although the position in which it is placed makes it look like a diamond and thus it gets its name.
18. The football field—is a rectangle upon which is placed many parallel lines which make small rectangles within the large one.
19. Chair on the floor—a chair which will not sit solidly on the floor indicates that the floor is not a plane (level) or that the ends of the chair legs are not in the same plane Four points are not necessarily all in the same plane.
20. The frame of a bicycle—a bicycle is made up of two or more triangles which make the frame rigid. A three sided figure cannot change its shape without breaking one of its parts.
21. Steering wheel on a car—a circle is used as a steering wheel on moving machines particularly automobiles in order that force can be applied in opposite directions to achieve the same purposes. The circle, furthermore, provides the opportunity to shift the application of the force easily and continuously.
22. The fence post brace—braces for fence posts are examples of the rigidity that can be introduced into situations by applying the triangle.
23. Making a straight row in the gar-

den—gardeners usually use two points with a string stretched between them agreeing with the principle that two points determine a straight line.

24. A tennis court—the tennis court is another example of a rectangle with parallel and perpendicular lines making up the interior markings.
25. Aeroplane three point landing—aeroplanes use three points in landing because three points determine a plane and therefore will rest solidly on any surface
26. Laying out a foundation for a house—strings or boards are used in laying out a foundation in such a way that they intersect at the corners. This is studied in geometry under the title of intersecting loci
27. Surveyors establishing a line—civil engineers use the principle that two points determine a line when they make their measurements for a road or a property line.
28. Laying a brick wall—bricklayers build up the corners or two points in a wall by means of their levels. By stretching a string between these two points, they determine the line for the remaining brick. Two points determine a line.
29. A railway track—the railway track is a very common example of parallel lines.
30. A tripod for cameras or surveyor's instruments—the tripod is preferred as a means of support for those articles that must be moved many times in the course of a small piece of work. Three points determine a plane and thus the article will rest solidly on practically any surface.
31. A telephone line and crossarm on the pole—in most cases the telephone wire meets the crossarm on the pole at right angles.
32. A rope fastened to a beam—since most beams are parallel to the surface of the earth and gravity pulls a freely swinging rope toward the center of the earth, a rope fastened to a beam illustrates two perpendicular lines.
33. A picture frame—there are more exceptions to this item than to some of the others but most everyone has seen a picture frame which is a rectangle.
34. A short cut across a vacant lot—the path across the corner is the peoples' way of making functional the fact that a straight line is the shortest distance between two points.
35. Ice cream cone—people of all ages enjoy a use to which the cone is put, namely, a recepticle for ice cream.
36. Three legged stool—three legs will fit more surfaces than four or more because three points determine their own plane.
37. Support for a shelf—a triangular form is usually used to support a shelf because of its greater rigidity.
38. Stove pipe—the stove pipe is an example of a cylinder.
39. A door—a door is usually rectangular in shape. Exceptions are some old style architecture church doors and doors in a few public buildings.
40. Placing the centerpiece on the table—efforts are usually such as to place the table decoration in the center of the table thus illustrating point symmetry.
41. The map of a state—maps illustrate similar figures with their proportional dimensions.
42. The shadow of a post—shadows are the real counterpart of the study of similar triangles.
43. The tracks of a sled in the snow—sled tracks are examples of parallel lines even when the lines are curved.
44. The sun is a circle in cross section. The rays of light from the sun are parallel because of the great distance between earth and sun.
45. The leaf of a book—leaves of books are rectangles.

46. The corner of a building—contractors are very careful that the corners of buildings are square (make angles of ninety degrees).
47. The shape of a room—most rooms can be classified as rectangular in shape.
48. A box—boxes are very good examples of rectangular prisms in solid geometry. They are sometimes given the special name of parallelopiped.
49. The curb of the street and the sidewalk—the relationship between side walk and street is one of parallel lines.
50. A star—a star as we see it without telescope, is a very beautiful illustration of the five sided figure, the pentagon.

These articles may be referred to definite geometric statements which are the theoretical basis of which the articles are practical illustrations. Following is a list of these geometric statements to which the articles in the above list are keyed by number.

a The shortest distance between two points is measured on the straight line connecting them. (34)

b Parallel lines are lines in the same plane that do not meet however far extended. (9, 18, 24, 29, 43, 44, 49)

c A rectangle is a four sided plane figure with its opposite sides equal and parallel and all of its angles right angles. (15 18 24, 33 39, 45, 47)

d A circle is the path of all points equal distant from a fixed point, the center. (10, 14, 21, 44)

e A pentagon is a plane figure of five sides. (50)

f A right angle is an angle of 90° or the angle formed by two lines which are perpendicular. (4, 24, 31, 32, 46)

g A rectangular prism is a prism whose base is a rectangle. (48)

h A cylinder is formed by a line which always moves parallel to its starting position and follows a closed plane curve which is called directrix. (38)

i A cone is formed when a line which always passes through a fixed point moves so as to follow a closed plane curve. (35)

j A plane is a surface such that a line joining any two of its points lies wholly in the surface. (3, 11)

k Similar figures are similar in shape. All dimensions are in the same ratio. (7, 12, 41, 42)

l Three points determine a plane. (19, 25, 36)

m A locus is a path of points such that all satisfying a certain set of conditions are on the locus and all points on the locus satisfy the conditions. (26)

n A square is a rectangle with all sides equal. (17)

o A line which touches each of two circles at only one point is a common tangent. (16)

q Symmetry is the concept of having a counterpart. There are three concepts to which symmetry is referred, the point called point symmetry, the line called line symmetry, and the plane called plane symmetry. (5, 6, 12, 13, 40)

r An isosceles triangle is a three sided plane figure with two of its sides equal. (8)

s If three sides of one triangle are equal respectively to three sides of another, the two triangles are congruent. (2, 20, 22, 30, 37)

t The formula relating to the total surface area and volume of right rectangular prisms and same concepts with reference to cylinders are:

Total surface area of a right rectangular prism $=(2l+2w)h+2lw$

l = length of base Volume $=lwh$

w = width of base

h = altitude of prism

Total surface area of a right circular cylinder $=2\pi r(h+r)$

Volume $=\pi r^2 h$ (1)

u Two points determine a line. (23, 27, 28)

Mathematics in CCC Education

By G. M. Ruch
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Purpose of the CCC—In any discussion of the educational program of the Civilian Conservation Corps it should be borne in mind that, as established in 1933, it had no primary educational objective in the sense of formal instruction. Rather, its purposes should be described as twofold: (a) work relief, and (b) conservation and development of natural resources.

This denial of educational objectives as a primary function of the camps should be tempered by two further observations. First, it should be pointed out that the first few months of operation of the program revealed the need for educational training, and that concrete instruction in the basic educational skills of language and mathematics was initiated very shortly.

Congress recognized this need in its Act of June 1937 extending the CCC for three years by restating the purpose of the camps in the following language:

> "for the purpose of providing employment, as well as vocational training for youthful citizens of the United States who are unemployed and in need of employment, and . . . for war veterans and Indians through the performance of useful public work in connection with the conservation and development of the natural resources of the United States."

The current Act, which was passed in August, 1939, extends the Corps through June 30, 1943 and also includes this statement just quoted.

In the second place, the educational value of work experience with its incidental vocational training can hardly be overemphasized in a day when a national emergency has forced the thinking of educators in the direction of vocational education to a degree unparalleled in the history of American education. William S. Knudsen, writing in *American Magazine* for June, 1939, prior to his appointment to the Council of National Defense, (now the OPM) has stated the case for work experience dramatically under the caption, "If I were 21." I quote several of his opening paragraphs.

> If I were twenty-one I would be a mechanic.
>
> I would try to get work in a machine shop. If that failed I would try for a job in a filling station, or as an apprentice to an electrician or a plumber, or as a clerk behind a counter, or as an errand boy.
>
> I would try to get some work to do with my hands.
>
> * * *
>
> Last summer I talked a good deal with some young college seniors. They all seemed worried about how they would get their start in life. "What shall we do?" they asked. "What shall we head for?"
>
> I suggested that they go out and try to get a job working with their hands: filling station, factory, store, machine shop. They were puzzled. They thought I was joking. Why, they were college men. Manual labor would involve (in some way I don't understand) a loss of prestige. Furthermore, these youngsters had never had any training in practical, mechanical skills. They wouldn't know how to start.

These brief remarks by Mr. Knudsen should help us to realize that the educational values of CCC education are of two kinds: (1) those eventuating from work experience, and (2) those resulting from formal classroom instruction. The present discussion, however, must be limited largely to the second of these types of values, and even more specifically to instruction in mathematics.

Administration of the CCC—Further perspective on the educational program of the work camps will be gained from a consideration of the rather unusual administrative organization of the Civilian Conservation Corps. This responsibility was assigned to the United States Army as the

sole governmental agency with any large experience in organizing camps. Because of the conservation aim of the work projects, the Federal departments of Agriculture and Interior logically assumed important roles in this program. And, somewhat later, the United States Office of Education was given the task of developing an educational program.

Although the educational program has come more and more to be recognized as a major function of the CCC, the following tabulation of the 35,391 employees of the Civilian Conservation Corps, as of August 1940, shows the divided and anomalous administrative set-up of the organization:

Employees of the Department of Agriculture	12,244	(34.6%)
Employees of the Department of the Interior	6,068	(17.1%)
War Department personnel (other than educational personnel)	14,630	(41.3%)
Educational personnel (employed by the War Department, but selected by the U. S. Office of Education)	1,572	(4.4%)
Employees in motor repair shops (operated in the field by the CCC director)	785	(2.2%)
Central administrative staff (including the CCC director and his appointees)	92	(0.3%)
	35,391	(99.9%)

It requires no comment to show that the almost universally admitted success of the CCC is to be ascribed to the splendid cooperation of a number of governmental agencies rather than to any logical or unified administrative structure. These figures also reflect something of the relative emphasis on the purely education function of the program when it is noted that, of the total personnel of 35,391, only 1,572 (or about 4.5%) represent educational personnel.

It has been suggested that the educational objectives of the CCC have assumed increasing importance during the lifetime of the camps. This is concretely shown by the assignment of the CCC to the Federal Security Agency under the first Federal reorganization act of 1939. This brings the CCC, the Office of Education, and the National Youth Administration into the same Federal agency, thus providing a better opportunity for coordinating the work of three Federal units fundamentally concerned with the welfare of youth. President Roosevelt, in his message of transmittal of the reorganization plan to Congress, clearly recognized that the educational function of the CCC transcends the work and relief functions when he said:

> Its major purpose is to promote the welfare and further the training of the individuals who make up the Corps, important as may be the construction work which they carry on so successfully.

The CCC Enrollees—There are now more than 1,350 CCC camps throughout the country with a total enrollment of more than a quarter-million boys and men. Since 1933 more than two and one-half million young men have been enrolled. The number of new enrollees annually is somewhat larger than the annual number of matriculates in our colleges and universities. The survival rate is approximately 80 per cent; that is, about 20 per cent, or 500,000 of the 2,500,000, left camp for reasons of discipline or desertion. Such a "mortality" rate compares very favorably with that of our colleges, particularly so when we allow for the fact that the change from home life to camp life with its regimentation is a far more drastic experience than is that afforded by the change from a high school to a college environment.

In order to build a picture in your minds of the typical CCC enrollee, the following ten statements are offered without much claim for completeness or orderly sequence:

1. The average age of enrollees is now approximately 18 years; the ages 17, 18,

and 19 now include more than three-fourths of the total.

2. His average height is 5 feet 8 inches and the average weight is 145 pounds.

3. He comes from a 5-child family, living in a 6-room non-modern house.

4. Two-thirds have had no work experience other than odd jobs about the home. Only one-fifth have had four or more months of employment.

5. His education is about equal to completion of elementary school; he has completed 8 plus grades in a little less than 11 years. Less than one-half of one per cent have had no formal education, about 14% are high school graduates, another 14% have attended college but only one-tenth of one per cent are college graduates.

6. About 37% of the enrollees are now taking academic studies in camp.

7. His parents have completed seven grades in school. They are typically American born although 20% are foreign born.

8. Ten per cent are Negroes.

9. Thirty-seven per cent come from broken homes.

10. His skill in reading and arithmetic is approximately that of the sixth-grade pupil. About one-fifth do not exceed the fourth-grade level.

The Mathematics Program in the Camps—The teaching of arithmetic in the CCC camps was originally undertaken with two groups of enrollees: (1) those who were illiterate and received primary instruction in the three R's; and (2) those who needed training in specific arithmetic operations, to supplement other training. The arithmetic taught to the illiterate consisted mainly of reading and writing numbers, and easy addition, subtraction, multiplication, and division, with perhaps some very elementary work with fractions. That given as supplementary or related instruction usually included only the work necessary for performance of the desired activity to which it was related.

It was early learned that, in many cases, enrollees who had had no previous elementary schooling, and those who had had little schooling, often were victims of circumstances, so to speak, in that they had been prohibited by the environment of their childhood days from receiving such schooling. Only occasionally would educationally backward enrollees be found mentally deficient. The average I.Q. of the illiterate or functionally illiterate enrollee was found, almost without exception, to be far greater than his public school grade level would indicate.

In camp the problem of offering instruction to enrollees deficient in elementary studies was complicated by the fact that each member of the company must take his place in the activities necessary for maintaining the CCC camp itself. This meant that instruction must be offered on the enrollee's time, and not during his regular work day. Although the average CCC enrollee has remained in camp only 9 months, those of lower academic level and those who have had little or no previous schooling have tended to remain for a longer period. However, since the maximum term of enrollment for any enrollee is only 2 years, it is necessary that whatever instruction is to be of benefit to such enrollees must be somewhat concentrated, by school standards, and must be presented in such an interesting manner that a greater portion will be retained than is usually the case with the public school child. This is especially true since the time afforded for instructional purposes in camp is also limited.

To meet the circumstances of the CCC camp situation, it was necessary for educational advisers and those in charge of instruction to determine just what objectives were most desirable, and to concentrate their efforts in working toward attainment of these objectives. For the most part such objectives were of practical nature; that is, they were selected so that their attainment might be of greatest practical value to the enrollee upon the completion of his CCC enrollment. For the illiterate, a desirable objective was usually considered the development of suf-

ficient skill in reading to enable him to read a newspaper, and in writing to enable him to write his own letters. For those who needed remedial work in elementary academic studies it was usually considered the attainment of an elementary school certificate or its equivalent.

Skilled instructors were not supplied the CCC camps. Therefore, the instructors had to be secured from whatever source possible, and trained to teach in the CCC organization. Officers and technical service foremen served as instructors; the educational adviser and his enrollee assistant did likewise; many times enrollees have been trained to teach. A few experienced teachers were obtained through WPA or through cooperative school projects. But the fact that instructors were not experienced as instructors did not necessarily mean that the quality of instruction was poor; these men all recognized the problems to be met in the camp, and they had the informational background necessary to enable them to teach the classes which they assumed. The greatest need was for instructional materials which could be used effectively in teaching in camp. Such material must of necessity be presented in more condensed form than that used in public school teaching; in order to be successfully mastered by the enrollee, it must be of specific interest to him.

Preparation of instructional materials to meet the needs of the CCC enrollee in his camp situation has been undertaken by the CCC Camp Division of the U. S. Office of Education, in the development of two series of ungraded text-workbooks: a six-unit series in language usage and a six-unit series in arithmetic. These series of workbooks are intended to carry an enrollee from very low elementary level to a point which may be considered the equivalent of elementary school accomplishment in these subjects.

In the arithmetic series, the first workbook is essentially an arithmetic reader. It deals with subject matter familiar to the CCC enrollee and is intended to familiarize him with numbers and basic arithmetic words. Calculations used in this workbook are extremely simple.

Workbook 2 in the arithmetic series takes up addition and subtraction. In this workbook all the addition and subtraction combinations are given. Lessons are developed with a sort of cumulative effect: that is, as the course proceeds, lesson material of previous lessons is studiously incorporated in exercises and materials. This cumulative effect is maintained throughout the remainder of the series.

Workbook 3 takes up multiplication and division; Workbook 4 introduces work with fractions; Workbook 5 gives additional work with fractions, including decimals.

The Camp Exchange affords an opportunity to introduce problems in buying and selling which are familiar to the enrollee. The camp workshop makes it possible to introduce problems in construction, in handling of building materials, and in reading blueprints and diagrams. The work project is a wide field of activity from which problems of many types may be selected. In the carry-over from the CCC camp to the enrollee's life as a private citizen is found the opportunity to consider banking, savings, and other social aspects of arithmetic.

Acknowledgments—Considerable portions of this paper are taken with minor changes from notes supplied to the writer by Dr. Ralph C. M. Flynt, Special Assistant to the Director of CCC Camp Education, U. S. Office of Education. The recent report on the Civilian Conservation Corps by the American Youth Commission of the American Council on Education has also been drawn upon freely without specific citation.

Service of Mathematics to Nurses*

By Adeline Hendricks
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Before talking directly on the subject assigned me, I believe I should briefly outline the growth of nursing education in the United States of America.

Nursing, as a profession, is the youngest of all the professions. Modern nursing was begun in Florence Nightingale's life time and her death occurred as recently as 1910.

The nursing organization, primarily interested in setting up standards for schools of nursing had its beginning in 1893. It is now called the National League of Nursing Education. At its first convention in 1894, a paper entitled "What is a Trained Nurse?" was read. It caused such discussion that immediate steps were taken to find out. We have been working ever since to find the answer to the question. In a constantly changing world, especially as far as medical science is concerned, I think that the question will never be fully answered, but each generation must answer it for itself as the nurses' work broadens from year to year.

The National League of Nursing Education gave to the state leagues in 1917 the "Standard Curriculum" which was followed in the schools of nursing until the revision was made in 1927, presenting fuller and richer courses of study. Ten years later in 1937, the National League of Nursing Education published "A Curriculum Guide for Schools of Nursing." This is not a revision. It presents an entire outline of nursing education. It will take years before we have a thorough understanding of it and before we can adjust nursing education to the standards presented in the newly arranged courses of study.

In the two former published curricula, a course called Drugs and Solutions or Elementary Materia Medica, a preliminary to Materia Medica Proper, was required. Of its disposition in the new curriculum, I will speak later.

The major part of the course in Drugs and Solutions presents to students, problems in the making of solutions and administering of drugs which they will need in their practical work as nurses.

The problems are based upon two systems of weights and measures, namely the apothecaries and the metric with the household equivalents for both systems. The student must know the tables of volume, capacity and weight in the metric system and those of weight and volume in the apothecaries system. She must be able to use the equivalents of one system in the other system and ordinary household measurement equivalents in either system where this is practical. For instance, one ordinary tumblerful approximately equals 240 cubic centimeters or eight ounces. One level teaspoonful is equivalent to one dram or four cubic centimeters.

Before teaching the student the actual preparation of solutions, she must understand what is meant by solute and solvent, and strength of a solution. She must be able to express the strength in the percentage method, the ratio method, or grams to an ounce method. A few very simple problems will illustrate the methods. For instance:

What is the percentage strength of a solution that contains two grams of boric acid in fifty cubic centimeters? Obviously, the answer is four per cent.

What is the ratio strength of a solution of bichloride of mercury made by dissolving a seven and one-half grain tablet in two hundred and fifty cubic centimeters of water? Seven and one-half grains equals five-tenths grams. Dividing two hundred

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and fifty by five-tenths we get five hundred. Therefore, expressed in the ratio method, this is a one to five hundred solution of bichloride of mercury.

The grains to the ounce method is simply an expression of the number of grains in each ounce of the solution.

Sometimes it is necessary to express the percentage strength in its equivalent ratio strength and vice versa.

When the student can readily ascertain strength of solutions, then she must learn to make solutions. We use three methods, namely:

1. Solutions made from powders or crude drugs.
2. Solutions made from stock solutions of known strength.
3. Solutions made from tablets.

May I illustrate each method by a very simple problem:

How much potassium chlorate would be necessary to make up one pint of a four per cent solution? If the student multiplies the desired quantity by the desired strength she will obtain the quantity to be used. One pint is equal to five hundred cubic centimeters. Five hundred times four hundredths equals twenty. Therefore, she will use twenty grams of the potassium chlorate.

The student may also work the problem by a proportion method, letting X equal the unknown quantity, and formulating the equation: $X:500::4:100$. When solved X equals twenty.

Another problem illustrates the second method: How much of a solution of silver nitrate containing forty-eight grains to the ounce, must be used to make up one pint of a one to one thousand solution? Since the forty-eight grains to the ounce represents forty-eight grains to four hundred and eighty minims, we have a ten per cent solution. The amount of the stock solution to be used can be found by dividing the strength of the desired solution by the strength of the stock solution and multiplying by the quantity to be prepared. Therefore, one-one thousandth divided by one-tenth times five hundred will give us five. The amount of the stock silver nitrate solution to use is five cubic centimeters.

To illustrate making a solution from tablets, suppose one hundred cubic centimeters of a one per cent solution of novocain is to be made from one-fourth grain tablets. One per cent of one hundred cubic centimeters is one cubic centimeter. This weighs one gram or approximately fifteen grains, which is the total amount of drug in the whole solution. If we are to use one-fourth grain tablets, we will divide fifteen by one-fourth to get the number of tablets. The answer will be sixty.

There are other types of problems that the nurse must know how to solve in order to calculate dosages. One that she uses very often is calculating fractional doses from tablets of known strength, such as the following example:

If we have on hand one-fourth grain tablets of morphine sulphate and are to give one-sixth grain, how will we proceed? Obviously, $X:\frac{1}{6}::1:\frac{1}{4}$, or divide the desired quantity by the quantity on hand and the answer will be two-thirds. We will use two-thirds of the one-fourth grain tablet. We will dissolve the one-fourth grain tablet in a certain quantity of water, say twenty-four minims and use sixteen minims of the solution which will contain the one-sixth grain of the drug.

Other problems used less frequently are calculating doses from stock solutions and calculation of doses for children from known adult doses.

All of these problems of which I have given you a few illustrations are worked either by simple arithmetic or the proportion method, where a simple equation with X as the unknown quantity must be solved.

The great difficulty that students have in this course is not that they cannot understand the reasoning involved in the solution of the problems but the majority cannot use the metric system. Some know nothing about the apothecaries system,

and very few can add, subtract, multiply and divide fractions and decimals and be sure that their answers are correct.

In the 1937 Curriculum Guide for Schools of Nursing the course of Drugs and Solutions does not exist as such but the knowledge to be acquired in the course is placed in several other courses with this explanation, and I quote from the Curriculum Guide:

"1. The curriculum committee has decided to omit arithmetic as a subject or as a definitely organized unit of study in any course. This is on the assumption that students have had or should be expected to acquire the necessary facts and skills in arithmetic before they enter the nursing school. The applications of arithmetic will be taken up in several courses.

2. It is strongly urged that applicants who have been accepted for admission to the school of nursing be sent information relative to the need for a careful arithmetic review before entrance to the school.

3. Sample arithmetic problems to guide the students review should be sent to the students with the suggested letter.

4. A pre-test in arithmetic should be given immediately upon the students entrance to the school; the passing grade should be seventy-five per cent.

5. In some situations where the pre-test indicates the need, it may be necessary to teach a unit of arithmetic to insure sufficient mastery in the application of arithmetical principle to special nursing problems relating to drugs and solutions.

In other situations, it may be sufficient to plan a special review or require tutorial aid for those who fail the pretest. It is suggested that a second test be given not later than six weeks after the student enters the school. It is suggested that failure to pass this re-test should disqualify the student from continuing the regular course in nursing."

If this method is followed, from the sample of the pre-tests and re-tests given in the Curriculum Guide, the student must know how to use the tables of measurements and be able to solve the problems of the types of which I have given illustrations.

In our own school of nursing, we have not yet discontinued the teaching of drugs and solutions as a separate course. Last fall we did, however, give a pre-test of the simplest sort. Two-fifths of the class did not attempt to solve the problems on changing per cent to ratio and ratio to per cent. Two students in the class had never learned anywhere, so they said, to use Roman numerals. (All prescription writing is done in Roman numerals.) One fourth of the class did not make a grade of seventy-five per cent. The majority could not solve a simple proportion problem. It is true that the class had not had an arithmetic review but the test was very elementary in comparison with the pre-test suggested by the Curriculum Guide.

I am not in the least objecting to the plan of the Curriculum Guide. I heartily agree that schools of nursing should not have to teach arithmetic and systems of measurement. But the stating of this fact does not solve the difficulty. I am sure that the students have been taught well and no doubt upon leaving the grades, could use arithmetic well.

How can the high schools help us in this problem? Our students come to us not from the grades but from high schools. Upon entering they have had one year of algebra, one of geometry, one of chemistry, and some have had one year of physics. It would seem that those courses should have helped the student to retain the knowledge of arithmetic, largely acquired in the grades.

We would like to use the suggestion of the Curriculum Guide and discontinue the teaching of drugs and solutions, which is largely a course in arithmetic, but if we do, the students coming to us from high schools must be able to use arithmetic and the metric and apothecaries systems, with a sense of surety.

In addition to the use of arithmetic in making solutions and calculating dosages, the nurse uses arithmetic in calculating certain diets for specific diseases, such as diabetis mellitus. The problem is to work out the amounts and kinds of foods that the patient may have, dependent upon the sugar content, and using the number of grams that will produce the calories prescribed and yet give a satisfying diet.

One of the debatable questions these days is, "Should the student contemplating nursing study geometry?"

In a textbook which our students use in surgical nursing, the author discusses dislocations, sprains, fractures, and their treatment. Under the title, "Position," he said this. "The fracture having been reduced, the part should be placed in the optimum position for retention and if the joint is involved, in the optimum position for ankylosis, much disability may be prevented by keeping joints that may become ankylosed in the position of greatest usefulness."

Then he suggests what to do with various joints:

"Shoulder joint: The arm should be treated in an abducted position at about fifty degrees with the elbow slightly in front of the plane of the body.

Elbow joint: An angle of about seventy degrees is the most useful position unless both elbows are involved when one should be fixed at an angle of about one hundred and ten degrees and the other at seventy degrees.

Ankle: The foot should be kept at right angles with the legs and in a slightly varus position. Ankylosis is least disabling with the leg in a very slight abduction, the thigh being extended and rotated slightly outward."

The student in orthopedic nursing will deal with weights and pulleys, traction and countertraction, terms such as plane, angle, rotation, arc, flexion, extension, and parallel. What do these mean to the student who has never studied geometry? Suppose a surgeon directs a nurse, in the treatment of an orthopedic patient, using any of these terms, as he often does. How intelligent will she be in carrying out the order if she does not understand the meaning of the terms?

Daily, the nurse helps to plot the temperature, pulse and respiration curve for every patient. If she has studied algebra, she will know what a curve means. Since in some diseases the curve is very definite, she will know when it is deviating from that expected and will look for symptoms of complications if she is intelligent about graphs.

I have confined this paper to mathematics and the student nurse. I have not touched upon the subject of mathematics and the graduate nurse. Her knowledge of mathematics, beyond that required of the student, will depend largely upon what she does as a graduate nurse.

Just as a suggestion, one of the questions in a final examination given this year to some graduate nurses who were studying Principles of Public Health Nursing was the following: "In a city of thirty thousand population, there were five hundred births last year. Indicate how you would find the birth rate. How does the birth rate compare with the rate for the United States?"

If the graduate becomes an executive in any field of work, the formulation of budgets, statistical reports and various types of studies will be a part of her duty.

The public health nurse must be able to interpret vital statistics if she is directing the work of a community.

The interpretation to boards and the public at large of any studies made may be by the use of various mathematical devices.

In closing, may I state again that the student comes to us graduated from high school. How can this young person come to us with the principles of mathematics and the sciences so thoroughly in mind that she can use them as ready tools in acquir-

ing fundamental principles in the science and art of nursing?

The superior nurse is the one who possessing the human characteristics necessary to the successful nurse can take all the principles that she has learned, whether from mathematics, physical or social sciences, or from allied arts, and apply them in curative nursing, prevention of disease, and the promotion of health in her community.

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The Mathematics Most Used in the Sciences of Physics, Chemistry, Engineering and Higher Mathematics

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As a student of mathematics in the secondary school, I accepted the teachings of my teachers as coming from individuals who were skilled in the ways of the world and knew just the things that were best for me to study. In those days, which are not so far past, the pupils did not question the wisdom of the teacher and stay in the class room to argue a point very long. Therefore the teacher was always right.

The things that we learned were presented from a textbook. This book had been wisely selected by this infallible teacher, and since the teacher had selected it everything in this book was of value and should be mastered by the student if he expected to pass the course, to graduate, and to continue his way into an institution of higher learning.

The assumption was, of course, that this textbook had been written by an individual who had gone through all the experiences of having attended a university, had specialized in the field of mathematics, and knew all about the topics that should be included in a course of secondary mathematics.

In due course of time I completed the mathematical training which was presented to me in the high school and entered a University. There some more courses in mathematics, and a few in physics were absorbed. In the years spent there, I began to wonder if my teachers really did know as much as I thought they did about the mathematics which was actually needed, in order to successfully carry the work presented in college, and also about the mathematics which was of little importance in the same endeavor.

Upon graduation from college, I found myself in the position of that teacher to whom pupils looked for guidance and who, having secured the training which was offered by one of the leading Universities of the country, was qualified, so my state certificate said, to teach the younger generation.

When face to face with this problem, I began to realize just how little I really did know about the things that should be emphasized in my own high school classes.

I knew that many of the things which were taught in the high school mathematics courses had not been used in my work in the university, also that the professors in the university assumed that I knew certain phases of mathematics with which I was not familiar. Therefore the question arose as to the abilities which should be emphasized in high school mathematics courses. Due to the economic necessity of making a living, I began to teach the various abilities stressed in textbooks, although I was not certain of the value of many of them. I wonder, how many other teachers have had, and are now having, the same experience. In the meantime, I began to seek the answer to the question of, what specific abilities should be emphasized in my mathematics courses.

The first method attempted was the question and answer method. I talked with other teachers of mathematics and tried to find out their views concerning the abilities that should be emphasized. Some were quite positive that they had the problem all solved. Others were more conservative and would merely venture an opinion that they thought that this or that topic was, or was not, important.

The main thing wrong with these answers was, that they did not agree. When I took my mathematics, most of my teachers were in agreement on this fact, that if several individuals worked the same problem their answers should check rather closely. This lead me to the decision to try another method of reaching a more satisfactory answer.

I looked next at textbooks to see if the authors, who are supposed to be specialists capable of putting together between the two covers of their books the facts that are essential to the subject field, could answer the question for me. I came to the conclusion that in most cases the authors were in a position where they had to make a book which would sell. In order to do this they attempted to cater to the divergent opinions of the various secondary mathematics teachers by presenting all the materials which they considered necessary.

This has resulted in textbooks which contain such a vast amount of information that it is virtually impossible for a teacher to conscientiously cover all of the material contained in most of the textbooks in the time traditionally assigned for the completion of the subject. It seemed rather obvious that there is a need for some objective data to be secured concerning the specific abilities which should be emphasized, to serve as a guide for teachers who wish to adequately prepare their students for continuance of scientific work.

Accordingly I decided to see if I could find definite data concerning the mathematical skills and abilities which should be developed by high school students who intend to study physics, chemistry, engineering, or higher mathematics. These four fields were chosen on the basis of a report of the Curriculum Committee of the Iowa Mathematics Teachers Association which set forth the necessity of a strong high school mathematics background for success in these courses.

Many studies have been made upon this question, in most of which textbooks have been analyzed by individuals, to find the skills which are needed to solve the problems in the texts investigated. The procedure used by most of these investigators has been to solve the problems in the texts, and then to tabulate the skills or abilities used. The results obtained in this manner may be influenced to a marked degree by the mathematical training and ability of the investigator, or by differences in the contents of the textbooks examined.

It was the purpose of my study to eliminate the individual mathematical ability of the investigator, and to extend the sampling of the textbooks examined. It is possible to secure more conclusive evidence concerning the mathematical abilities which should be emphasized by teachers of secondary mathematics, by assembling data from all of the available studies, and then determining the actual skills or abilities which have been used most often by the investigators.

Due to the fact that it is not practical for teachers of secondary mathematics to prepare their students for any one science, no attempt was made in this study to distinguish the mathematical abilities needed for each science.

The Bibliography of Research Studies in Education, and the University of Iowa Card catalogue were searched for all studies listed under the headings of mathematics, physics, chemistry and engineering. Whenever a title gave indication that it concerned the mathematics used in one of these sciences, it was included in the list of theses to be analyzed. Whenever an additional investigation was mentioned in one of the studies being examined, the subject of which appeared to justify its inclusion in this study, it was added to the list to be collected. The detailed lists of skills prepared by Mark J. Flanders, Helen Q. Scribner, and Paul B. Sharer were used, and tables providing opportunity to list the frequencies of these skills were made. Other skills were added as they were found. When actual listing of the frequencies was begun, it was found that only the

seven studies which analyzed college physics textbooks, sponsored by Dr. C. J. Lapp, of the University of Iowa, could be listed on these tables. Since it was the purpose of this study to determine objectively the specific abilities to be emphasized, it was decided to place these seven studies together as one part of the study, and to place summaries of twenty-four other studies, which were secured, in a second part.

As it was not possible to make a single table which would include the findings of all the theses analyzed, the objective conclusions reached in this study, from comparisons of the seven investigations, were materially strengthened by the findings of this large number of other investigations, necessarily rather subjectively examined.

The higher frequency skills of the seven studies were carefully examined, and the detailed skills were grouped together thus securing a list of more generalized abilities which were important in the solutions of the problems of the college physics textbooks analyzed.

To support this list, and to supplement it, tables of the abilities which were mentioned as important by the twenty-four investigators were made and examined. If an ability was mentioned as important by two or more of the investigators, it was placed in a list of abilities which were important in the solutions of problems in the several sciences investigated.

If an ability appeared in both of the lists of important abilities, or if the supporting evidence from the information collected was strong enough, the ability was then placed in the list of abilities which should be emphasized by teachers of secondary mathematics.

This list of abilities has been secured by a careful analysis of thirty-one separate investigations which have been conducted at ten different institutions of higher learning. Twenty-eight of the investigations are Masters' theses, and three are Doctors' dissertations.

If an ability is not mentioned in this list, it does not necessarily mean that it should not be taught. There may be other reasons for including it in a high school mathematics course.

A brief summary of the evidence which justified the inclusion of each ability which this study has shown to be important in the sciences examined follows:

Algebra

1. Use formulas, selecting the proper formula, and making substitutions in it.

The use of the formula was found to be an important ability by nine separate investigators, three of whom found it necessary to select the correct formula 2022 times in solving the problems in three different college physics textbooks.

2. Solve linear equations.

The solutions of linear equations was found to be important by nine investigators. Seven of these found that it was necessary to solve linear equations a total of 4892 times in the textbooks which they examined.

3. Use ratio, proportion and variation.

While no frequencies could be definitely assigned to this ability, there were ten different authors who stated in the conclusions reached from their studies that use of proportion was important.

4. Read and make graphs.

Eight separate authors found this ability to be important.

5. Make algebraic substitutions.

Twelve different authors found this ability to be important. The ability possessed the highest frequencies of any ability tabulated by the seven investigators of college physics textbooks, who found it necessary to make a total of more than 14,000 algebraic substitutions of different kinds in solving the problems in the seven textbooks that they examined. The skill of, "Substitute a positive pure number for a positive algebraic number," alone accounted for 8224 of these substitutions.

6. Multiply and divide monomials.

This ability was found to be important

by seven investigators. The two skills which had the highest frequencies being: (1) "Divide a positive algebraic number by a positive algebraic number," with a frequency of 1564, and, (2) "Multiply a positive algebraic number by a positive algebraic fraction," with a frequency of 648.

7. Solve quadratic equations.

Ten investigators found that this ability was important. Seven of these had to solve 390 quadratic equations to work the problems in their books.

8. Solve simultaneous equations.

This ability was found to be important by seven investigators. Five of these had to solve simultaneous equations a total of 73 times.

9. Square a quantity.

This ability was used by ten investigators. Three of these said that the ability was an important one. The other seven used a skill which was stated as "Raise both sides of an equation to a power" a total of 61 times. The fact that the example given for this skill was $M^2=G^2V^2$ leads one to believe that squaring was most often used, although the actual evidence is weak on this point.

10. Make simple applications of parentheses.

Nine investigators found it necessary to make simple applications of the parenthesis. Seven of these made use of parentheses a total of 358 times.

11. Form an equation in solving verbal problems.

The formation of equations was found to be important by five investigators. Two of these said that it was important and the other three found it necessary to form equations a total of 1073 times.

12. Use exponents.

Seven investigators made use of exponents. Two of these said that an ability to use them was important. While the frequencies of actual use were not found to be very high by the other investigators. The fact that five of them made use of exponents seems to justify this ability in the list of those to be emphasized.

Geometry

1. Make scale drawings.

This ability was found to be important by ten of the investigators.Three of these mentioned that the ability was important, the other seven found it necessary to draw a line to scale a total of 568 times.

2. Compute linear measure.

Eight investigators found this ability to be important. Seven of these had to measure a distance a total of 485 times.

3. Construct a right angle.

Seven investigators made use of the right angle construction a total of 173 times, and in addition, five of these also made use of the construction of a line through an external point, perpendicular to a given line, a total of 68 times.

4. Compute areas and volumes of the common geometric figures.

Three investigators found this ability to be important.

5. Construct an angle by means of a protractor.

Seven investigators did this a total of 149 times.

6. Apply the Pythagorean theorem.

One investigator found that the ability to use the right triangle in computation was important. Six others applied the Pythagorean theorem a total of 51 times.

7. Draw a line through a given point parallel to a given line.

This ability was found by seven investigators a total of 120 times.

Arithmetic

1. Use the four fundamental operations for integers, fractions and decimals.

This general ability embraces such a large number of skills that space does not allow detailed evidence concerning all of them. The fundamental operations for integers possess the greatest amount of supporting evidence of importance, with multiplication and division being mentioned as important more often than addition and subtraction. At least twenty investigators of the thirty-one found the use of multiplication and division of integers to be im-

portant, and at least sixteen found the addition and subtraction of integers necessary. Decimals also were found to be important more often than common fractions.

2. Convert fractions to decimals.

Two investigators mentioned that this ability was important. Six other investigators changed proper fractions to decimals a total of 650 times.

3. Use percentage.

Five investigators found this ability to be important. The lists of skills used in the seven studies were so detailed, that it was not possible to tell which of the specific skills might be applied to percentage and which to some other ability.

4. Square a number.

This ability was mentioned as important by only one investigator, but all seven of the investigators who made the detailed studies found the ability. The total frequencies were over 900 times, so it was thought advisable to place this ability in this list.

5. Extract the square root of a number.

This ability was used by seven investigators a total of more than 250 times.

6. Change fractions to similar fractions or to whole numbers.

This ability was found to be important by eight investigators. Of these eight, the seven who made the study by tabulating the detailed skills were very consistent in using skills pertaining to this ability. Improper fractions were changed to whole numbers a total of 1440 times, and fractions were changed to simplest forms a total of 256 times.

7. Make conversions from exponents to ciphers, and from ciphers to exponents.

All seven of the investigators who made the detailed studies found skills related to this ability. Ciphers were changed to a positive exponent 285 times; decimals were changed to ten raised to a negative power, 123 times; and ten raised to a positive power was changed to a figure with ciphers a total of ninety-one times.

8. Find the least common denominator.

Seven investigators found need for this ability 118 times in their studies.

9. Locate the decimal point accurately.

Two investigators stated that this ability was important. The seven investigators who used detailed skills, did not isolate this one. They did, however, imply its use through the use of the fundamental abilities with decimals.

10. Use denominate numbers.

The ability as stated here is the summation of a wide variety of statements concerning denominate numbers. Fourteen different statements were made by investigators concerning the importance of denominate numbers.

Trigonometry

1. Make simple applications of the sine, cosine and tangent.

Most of the usages of these trigonometric functions were concerned with using tables to find the values of the functions when the angle was known in whole degrees. There is very little evidence to indicate that the process of interpolation should be taught in high school.

This study has shown that algebra and arithmetic play a more important role in the study of the sciences, than does geometry. It also shows that a thorough understanding of the simpler aspects of trigonometry should be developed.

Mathematics is essential for the adequate training of students in the scientific fields. Often students who enter the scientific fields seem to lack the ability to handle the mathematics involved. Science instructors seldom ask that less mathematics be taught, rather they hold that their students are not adequately prepared I quote from an article in the *Daily Iowan* for June 22, 1940. This quotation is taken from an interview with Dean F M. Dawson, of the University of Iowa's college of engineering, in which he announced that the United States commissioner of education had been offered the facilities of the college for training 200 stu-

dents in specialized courses, stressing mechanical engineering for national defense. Dean Dawson says, "Lack of sufficient training in simple algebra and trigonometry is the largest detriment to teaching students fundamentals of mechanical operation and aerial navigation."

Dr. C. J. Lapp, Physicist at the University of Iowa speaks also of the mathematical deficiencies of the students who come to his classes. He says, however, that he believes that the high school teachers, in general, are doing a good job of teaching mathematics. The main trouble, according to Dr. Lapp, is more fundamental, and is concerned with the psychology of transfer. He has said that the trouble is that we are not giving the student a basket in which to carry his mathematics from the mathematics room to the science room.

This gives rise to a consideration of the possibility of a more effective presentation of high school mathematics to students who intend to enter college in order that they may be able to make this subject function when the occasion arises.

The study which has just been discussed indicates that we do not need to teach more mathematics in order to prepare students to enter scientific studies. The abilities reported as being important are included in most present day algebra courses. I suggest, therefore, that we have a revision of the placement of the mathematics courses offered.

The "basket" of which Dr. Lapp speaks is an appropriate term. I disagree with him, however, that we are not giving this basket to the pupil. He overlooks the characteristics of baskets. They usually are open at the top, and are not very satisfactory for the purpose of holding liquid substances. Acquired knowledge is rather a liquid thing. It evaporates with time and leaks from this basket.

Are we not, therefore, expecting the pupil to retain the knowledge which he learns in the traditional school in grades nine, ten, and sometimes eleven over quite a long period of time when in many cases it is not used until two or three years later? How many people remember the license number of their cars two years after they are used? How many remember their old telephone numbers two years after a change? I find myself at a loss to recall the car license that I gave up last January.

Let us reduce the length of time from the studying of mathematics to its use by a revision of grade placement.

The following plan was suggested by Dr. H. R. Douglass in his lecture, "Secondary Education for the Other Half." He suggested that students not planning to go to college should take one semester of mathematics each year in high school, this course to deal with mathematical situations encountered in every-day life.

If this plan of yearly mathematics is recommended for these pupils, why not use a similar schedule for academic mathematics for those intending to go to college?

May I present the following plan?

9th grade—one semester—The first half year of algebra should be presented as a required subject.

10th grade—one semester—The first half year of plane geometry should be presented as a required subject.

11th grade—1st semester—The second half year of algebra should be presented as a required subject

11th grade—2nd semester—Third semester algebra should be presented as an elective.

12th grade—1st semester—The second half year of plane geometry plus some algebra and simple trigonometry should be presented as a required subject.

12th grade—2nd semester—Solid geometry with some algebra and trigonometry should be presented as an elective.

This plan presents the easier phases of both algebra and geometry to the student in his earlier high school years, leaving the more difficult portions of secondary mathematics to be taken when his mind is more mature, while still presenting enough

mathematics to the student who chooses to take high school chemistry or physics. The investigators are rather consistent in agreeing that not much algebra is used in chemistry, so chemistry might well be taught in the 11th grade, where it is already placed in many secondary schools. Physics might well be deferred until the 12th grade, thus allowing the student the advantage of all three semesters of algebra, if he wishes to take them.

Such a course would meet entrance requirements for colleges still having specific mathematics entrance requirements. It does not crowd other courses from the curriculum. Students who wish to take advanced algebra and solid geometry may do so electively. The abilities developed in the earlier courses may be maintained through the entire high school course rather than allowed to disappear to such an extent during the latter years of high school that they appear to be non-existent when the pupil enters his science courses in college.

Let us teach our mathematics in such a way that it will be a functioning ability when needed for further scientific study.

The author is interested in curriculum problems in mathematics and would enjoy corresponding with others of similar interests.

The list of abilities which follow has been secured by a careful analysis of thirty-one separate investigations which have been conducted at ten different institutions of higher learning. Twenty-eight of the investigations were master's theses and three were doctor's dissertations.

If an ability is not mentioned in this list, it does not necessarily mean that it should not be taught. There may be other reasons for including it in the course of study.

It is recommended that the following abilities receive special attention in high school mathematics courses:

Algebra

1. Use formulas, selecting the proper formula and making substitutions in it
2. Solve linear equations
3. Use ratio, proportion and variation
4. Read and make graphs
5. Make algebraic substitutions
6. Multiply and divide monomials
7. Solve quadratic equations
8. Solve simultaneous equations
9. Square a quantity
10. Make simple applications of parentheses
11. Form an equation in solving verbal problems
12. Use exponents

Geometry

1. Make scale drawings
2. Compute linear measure
3. Construct a right angle
4. Compute areas and volumes of the common geometric figures
5. Construct an angle by means of a protractor
6. Apply the Pythagorean theorem
7. Draw a line through a given point parallel to a given line

Arithmetic

1. Use the four fundamental operations for integers, fractions and decimals
2. Convert fractions to decimals
3. Use percentage
4. Square a number
5. Extract the square root of a number
6. Change fractions to similar fractions or to whole numbers
7. Make conversions from exponents to ciphers, and from ciphers to exponents
8. Find the least common denominator
9. Locate the decimal point accurately
10. Use denominate numbers in both the English and the metric systems

Trigonometry

1. Make simple application of the sine, cosine and tangent

◆ THE ART OF TEACHING ◆

An Experiment in the Use of Graph Papers

By Norma Sleight
New Trier Township High School, Winnetka, Illinois

Representing statistical data by means of the graph plays such an important role in our present day world that it behooves us as the teachers of mathematics to investigate with our classes the uses and misuses of this powerful tool. The pupils come to their junior year well acquainted with the procedures for making simple statistical graphs but with little knowledge of their real uses. The interpretation of circular graphs is very clear to them because they are simple and they have seen so many of them in various publications, but the possibilities of the line graph seem to be vague. They interpolate where they shouldn't, and extrapolation is a new word to them.

So this year a sample of nearly every kind of graph paper produced by a well known company was purchased, and I was amazed to learn of the varieties and uses made of this simple discovery by Rene Descartes. These papers were thumbtacked on the bulletin boards in the classroom; the wall was literally papered, because there were 59 varieties. Before discussing the uses made of this material, an outline of the types will be given.

There are a dozen general papers with main divisions as well as subdivisions marked in squares. Thirteen more rectilinear charts were made for specific purposes, on some the main divisions are squares with rectangular subdivisions; others are divided into rectangles and the subdivisions are squares. A few titles will give an idea as to uses made of these graph sheets. One, "6×8 divisions per unit—Security Prices, 27 weeks." The six divisions per unit on one axis represent 6 business days per week and each of the eight divisions per unit in the other direction stands for $\frac{1}{8}$ of a point rise or fall. Some other titles are "One Day by Hours," "One Month by Days," "One Year by Days," "One Year by Months," "Five Years by Months," etc. These are well labeled on the margins. For example, the last mentioned has 5 cycles of the 12 months by name and a blank for each year on one axis, and 0 to 100 on the other.

There are 16 types of general semi-logarithmic papers, from one to five cycles, some arranged horizontally, others vertically on the paper. A 17th type is a three cycle semilog sheet with "Five Years by Months" as a title. The longer axis is the logarithmic and the shorter is divided into five main divisions, each subdivided into twelve parts with months marked. There are five logarithmic papers with combinations of number of cycles from 1×1 to 3×3.

The remaining sheets are the most spectacular of the group. One is a "Circular Percentage" marked every .5%. Another, "Triangular Co-Ordinate" paper. This is a large equilateral triangle, divided into many smaller ones, 100 divisions to each altitude. This can be used for things or facts composed of three elements whose sum is constant, such as alloys containing three metals, chemical compounds, concrete mixtures, etc. This can be done since the sum of the three distances to the sides of an equilateral triangle from any point within equals the altitude—or 100%. There are two types of Polar Co-Ordinate paper. The first has degrees marked on it, so can be used for circular charts as well as to shorten surveying calculations. The second type has a rectilinear chart super-

imposed on the Polar and has a very specific use. It is called Fluxolite and is trade marked. This is used for rapid calculations on problems of illumination. A "Power-Emission" paper is constructed for the Institute of Radio Engineers for plotting the relation between filament power of vacuum tubes and emission current by extrapolation. It is logarithmic and the horizontal lines curve downward gently to the right. Another sheet used by the same organization is the "Radio Receiver Performance" paper. There are two separate sections on this paper. The upper part is labeled Fidelity and is logarithmic with axes marked *Frequency* and *Output Audio Voltage*—the lower, Sensitivity, is semi-logarithmic, *Frequencies* being on the uniform scale and *Input in Microvolts* on the logarithmic.

Individual companies have graph sheets made for specific purposes. Bell Laboratories have a "Standard Graph Sheet, Reactance-Frequency." This paper is "especially designated to solve quickly the relationships between reactance, capacitance, inductance, and frequency." It is logarithmic in four ways. The vertical and horizontal scales are logarithmic, being 8×6 cycles, then at an angle of 45 degrees are two more scales, logarithmic, cutting diagonally across the paper both ways. The result is weird but interesting.

There is a township sheet having miles numbered and showing subdivisions into sections; a traverse sheet for latitudes and courses. The "Reciprocal Ruling or Hyperbolic" sheet looks like a semi-log paper at first glance but is not. The one axis runs from 0 to ∞ the spaces being proportional to the reciprocals of numbers; the spaces on the other axis being evenly divided. This graph is useful for plotting gas or electric rates. Because of the changes of rates at various levels of consumption, the ordinary paper gives as a graph an irregular curve but the reciprocal paper produces a broken line, all sections being straight segments. Rates can be compared more easily and prices determined more readily from this graph. "Isometric" paper is a sheet covered with tiny equilateral triangles. Three dimensional objects can be outlined on this paper, the drawing being to the same scale in all three directions. The result is unnatural in appearance because proper perspective is lacking, but details are accurate for the dimensions along three axes. These pictures make it easy to explain machine details to those who are confused by mechanical drawings of three views.

The next and last sheet is for perspective drawing. It appears to be a hollow rectangular block, $12\times12\times8$, and divided into transparent cubes. It has two vanishing points so that if drawings of objects are made on it following the lines, the illustrations present a natural aspect. Because the guiding lines are blue, a photograph of the drawing does not show the guiding lines of the paper but leaves a drawing standing out in perspective.

The remaining part of this article will be devoted mainly to a report of the uses and results obtained from this exhibit. Frankly these 5 dozen $8\frac{1}{2}''$ by $11''$ graph sheets were placed on the walls more as a stunt than for any other reason but after a short time I began to realize that I had a powerful means of motivation and integration. Only the surface has been scratched, I am certain. At present there are* two algebra classes for which I am responsible, one beginning and one advanced. I did not think the beginning class would be interested and had no intention of doing much with the exhibit for them. But the first day the paper was posted, when I came into the room I discovered the entire class, faces to the wall, trying to read the titles in fine print on each sheet. As a class they had read the chapter on Rene Descartes in *Men of Mathematics* by E. T. Bell, and other references on his life. They were familiar with both graphic and algebraic solutions of two linear equations in two unknowns. They knew about X and Y in-

* Both are accelerated classes, being composed of the upper 10%.

tercepts and how to find the slope of a line graphically and by throwing the equation into the slope form, $y=mx+b$ and reading m, so they were not without background. They demanded to know the purpose of the different types, choosing the unusual looking ones of course. We discussed Polar Co-Ordinates right then and there just a short time after they had been introduced to rectangular Co-Ordinates. They seemed satisfied that points could be located uniquely by this scheme as well as by the other. The explanation of the advantages of employing Polar Co-Ordinates at times was entirely unrehearsed so it was a bit difficult to come down to their level of vocabulary and knowledge of Mathematics. Drawing maps of elevations seemed to be the best way out and this brought in a bit of trigonometry with the fact that a right triangle is determined when its hypotenuse and an acute angle are known.

We discussed commercial uses of graphs, and tried a few, plotting tables of numbers, then drawing the straight line which followed most nearly the trend of points. Quite heated arguments followed when it came to drawing the line, but they were satisfied to accept an approximate position of it when informed that there were ways for them to find and place a line a bit more accurately when they had had more algebra. Instead of examining $y=mx+b$ abstractly, then, it was done concretely with sets of data that sounded "lifelike." Given two points or one point and the slope, they were able to write the equation of the line. Generally, with these empirical equations, two points were chosen. Extrapolation from the written equation took on a real meaning because results were checked on the graph. The two boys who disagreed most violently found that their results from extrapolation gave only a very small fraction of 1% variation which quieted them somewhat. One boy whose mother is employed in the office of the Visiting Nurses' Association in a town nearby, collected data for the expense of running this organization for the past 5 years, plotted the points, drew the trend line, took two points on it, then estimated the cost of 1940, by writing the equation of the line and substituting 6 for X. He says his mother points with pride to that chart which hangs on the wall in her office. Will the reader please remember that this is a selected group but in my opinion it is about time that we turned our attention and efforts to finding something challenging to the gifted.

The semi-log paper interested this group, perhaps because there are so many kinds. Now logarithms and this type of graphing is quite a long way from one semester of algebra so I told them they were too young. That was the wrong (or right?) reply. We spent two days on the use of this type of graph. Professor Hogben's little table of exponents came to our aid:

2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9...
2	4	8	16	32	64	128	256	512...

We defined and examined arithmetic and geometric progressions, positive exponent, integral and decimal. The two types of series are illustrated in the above table. We multiplied numbers to the base 2 by adding the logarithms (exponents). The C and D scales of a demonstration slide rule were studied and we even did a little multiplying, factors being chosen so that too many questions would not arise to cloud the issue. Squaring a number doubled the space on the rule as well as on the graph, while cubing a number tripled the space. Following the study of the construction of the semi-log paper just one example was given to show the commercial use made of log graphs. Since a percentage gain year by year is a geometric progression with r equal to a fraction slightly improper, if percentage gains are equal, lines will have the same slope (be parallel) on the semi-log graphs no matter how great the actual differences are. Here the class dropped behind so we made up a problem like the following and graphed it on the regular paper and the semi-log paper:

Two small business firms, A and B for short, made profits on their investments for three years as follows:

	Capi-tal	*1st year*	*2nd year*	*3rd year*	
A	$1000	1080	1166.40	1259.712	8% increase
B	$ 500	550	605	665.50	10% increase

The reader knows what happened with these data. On regular paper we have two curves working upward, A faster than B, but on the other sclae they form straight lines and the slope of B is the more abrupt. This procedure produced satisfaction. We pointed out the unfairness of comparing the profits say of a branch store with the main one except on the semi-log paper because of the unequal bases.

Now these beginners at this stage in their development cannot be expected to remember all that we examined in these few class periods but I believe that there are two phases in the studying of mathematics (1) complete mastery of certain topics and (2) an appreciation of the beauty, consistency, structure, and possible uses of Mathematics. It is to the latter aim that the array of graph papers makes the most significant contribution.

Now for the juniors. Nearly all the types of graph sheets were inspected by them and we were not so hampered in explanations because they had had quadratics, progressions, and logarithms. They had graphed simple and compound interest at various rates for a period of 20 years on the same axes. $S=\frac{1}{2}gt^2$ had been graphed. So with this group the uses of the semi-logarithmic and logarithmic graphs were studied with ease. They finally knew that a formula showing percentage gain such as the compound interest or population formulas, i.e., any equation of the form $y=ab^n$, produces a straight line on the semi-logarithmic paper, also an equation of the form $y=ax^b$ produces a straight line on the logarithmic paper. They were also pleased to find that only two points were necessary to establish the graphs of the curves represented by such equations as $s=\frac{1}{2}gt^2$ and $A=\pi r^2$ if logarithmic paper is used. To some of them it was quite a revelation, since they had found it necessary in their previous experience to use so many points. One girl in the class went to the Superintendent's office, secured enrollment data of our school from 1926 to 1939, graphed it on Cartesian paper, then on semi-log paper. The first graph increased its upward curvature definitely at the right, the second bent away from its upward path, the direction being nearer the horizontal. Both showed increases as the slope was positive. Again this afforded an excellent chance for discussion of increment of change versus rate of change. Definitely the per cent gain in population at the school is dwindling. The junior group was very much impressed with the size of numbers that can be plotted on log sheets. For example, if a semi-log is three cycle, the first third of the axis may be 100, then the end of the second cycle is 100^2 or 10000 and the third is 100^3 or 1000000. Regular paper has definite limits.

Surely some of this material can be used in teaching geometry, physics, and in commercial subjects. The "Isometric" and "Triangular Co-Ordinate" and "Perspective" look good for material in geometry. The above report is based wholly upon experience and I wish to limit the discussion to that. To sum up, if one wants an easy interesting helper in putting over some modern uses of mathematics, this experiment affords an opportunity with very little effort and great effectiveness. The pupils enjoyed it, and so did I.

Note: While 59 varieties of graph paper make quite an impression, 12 or 15 carefully selected would be sufficient to carry out the work. Keuffel and Esser has a catalog entitled "Graph Sheets, Co-ordinate Papers and Cloths with illustrations of Their Use." This little booklet proved very helpful at times because it has clear directions for the uses of the more unusual forms of graph paper. Other companies doubtless have contributions they could make.

EDITORIALS

The Importance of Mathematics in the War Effort

THE FOLLOWING correspondence and the accompanying comments will be of interest to all readers of THE MATHEMATICS TEACHER and those who appreciate the importance of a good mathematical background in the training of all American citizens. It should be read by those school officials who in the recent past have been preaching the elimination of mathematics from the curricula of secondary schools:

UNIVERSITY OF MICHIGAN
ANN ARBOR, MICHIGAN
Bureau of Cooperation with Educational Institutions

THE NAVY AND MATHEMATICS

Stop! Look! Read! and then cogitate on the meaning of the two letters herein reproduced. These letters started some interesting discussions at the Principals' Meeting in Lansing, December 4 and 5, as many of you will recall. It is thought that many others will be interested.

Captain F. U. Lake
Head of the Training Division
Bureau of Navigation, Washington, D. C.
My dear Captain Lake:

When Admiral Nimitz visited the campus of the University of Michigan the other day, he mentioned that there had been some difficulty in finding students in American colleges other than engineering who were sufficiently prepared in mathematics to make them available for training for commissions in the Navy. This situation ought to be called to the attention of educators in colleges and secondary schools throughout the country. I should deeply appreciate receiving a statement from you on this matter, especially if you could give me such facts and figures as would constitute a self-evident argument. I hope also that it will not be necessary to set any restrictions on the use of such information. It seems to me that educators should promptly recognize the danger, if there is any, from our past softening of our educational programs.

Very truly yours,
LOUIS I. BREDVOLD
Member of the University Advisory
Committee on Military Affairs

My dear Professor Bredvold:

Thank you for your letter of October 30. While we have not felt that it was our business to compile exhaustive data on our observations of the products of the educational systems of this country, we are in a position to give you some information on this subject.

A carefully prepared selective examination was given to 4,200 entering freshmen at 27 of the leading universities and colleges of the United States. Sixty-eight per cent of the men taking this examination were unable to pass the arithmetical reasoning test. Sixty-two per cent failed the whole test, which included also arithmetical combinations, vocabulary, and spatial relations. The majority of failures were not merely borderline, but were far below passing grade. Of the 4,200 entering freshmen who wished to enter the Naval Reserve Officers' Training Corps, only 10% had already taken elementary trigonometry in the high schools from which they had graduated. Only 23% of the 4,200 had taken more than one and a half years of mathematics in high school.

This same lack of fundamental education presented and continues to present a major obstacle in the selection and training of midshipmen for commissioning as ensigns, V-7. Of 8,000 applicants—all college graduates—some 3,000 had to be rejected because they had had no mathematics or insufficient mathematics at college nor had they ever taken plane trigonometry. Almost 40% of the college graduates applying for commissioning had not in the course of their education taken this essential mathematics course.

The experience which the Navy has had in attempting to teach navigation in the Naval Reserve Officers' Training Corps Units and in the Naval Reserve Midshipmen Training Program (V-7) indicates that 75% of the failures in the study of navigation must be attributed to the lack of adequate knowledge of mathematics. Since mathematics is also necessary in fire control and in many other vital branches of the naval officer's profession, it can readily be understood that a candidate for training for a commission in the Naval Reserve cannot be regarded as good material unless he has taken sufficient mathematics.

The Navy depends for its efficiency upon trained men. The men are trained at schools conducted for this purpose and the admission of men to these schools is based upon the meeting of certain carefully established requirements. However, in order to enroll the necessary number of men in the training schools, it was found necessary at one of the training stations to lower the standards in 50% of the admissions. This necessity is attributed to a deficiency in the early educations of the men involved. The requirements had to be lowered in the field of arithmetical attainment. Relative to the results obtained in the General Classification Test, the

lowest category of achievement was in arithmetic.

A study has been made of the grades received in the examinations of candidates for enlistment in the Navy, classified geographically according to the location of the recruiting station through which the candidates applied for enlistment. It is to be noted that the proficiency in arithmetic in the eastern part of the country was strikingly greater than that of the middle west and west. The lowest average mark east of the Mississippi was equal to the highest average mark west of the Mississippi. The three highest average attainments in arithmetic were achieved by the recruiting stations in Troy, Brooklyn, and Buffalo—all in New York State.

May I express the hope that this information will be of assistance to you.

Sincerely yours,
C. W. Nimmitz,
Chief of Bureau.
(Signed) F. U. Lake,
By direction.

When secondary schools eliminate not only trigonometry but also algebra and geometry from their programs, and then most of the reasoning problems of arithmetic, since pupils say they are too difficult, and offer as substitutes general mathematics in the ninth grade, social mathematics in the tenth grade, and review of arithmetic in the eleventh or twelfth grade as the total mathematical program of the school, where along the educational ladder are pupils to obtain experience in reasoning and in practice in solving progressively more difficult mathematical problems? Where in the course of the four years are youth to find mathematical problems which will extend their intellectual horizons and stretch their mental muscles?

George E. Carrothers
Director, Bureau of Co-operation
December 13, 1941

Teachers and administrators alike should begin now to reorganize the mathematics of the schools not only to meet the demands of the emergency but also to enable us to solve the important problems of peace that lie ahead.

W.D.R.

Standards for Mathematics Teachers

There is now a definite shortage of teachers all over the country. Standards for teachers of mathematics in some parts of the nation have never been satisfactory. It will be a temptation now for school officials to employ people to teach mathematics who know very little about the subject, or who are not more capable intellectually to pursue the study of the subject than some of the inferior types of teachers we have had saddled on us in the past. Teachers of mathematics, particularly heads of departments, where they exist, and others in positions of influence should do all they can to hold up standards throughout the present emergency, so that when the war is over our children may still have competent teachers to lead them.

W. D. R.

The Textbook Outlook

The following material collected by people interested in education will bear careful study. The shortage of textbooks in some places is very acute and school officials need the latest information on this matter.

Rumors of impending shortages in all sorts of materials are flying about these days. Some are undeniably true; some have little or no foundation.

Numerous enquiries have been made concerning the possibility of a shortage of book paper. Instances have been reported of some school executives who, hearing that new textbooks might be hard to procure in 1942, have had expensive repairs made on old books at a cost approaching the price of new editions of the same books.

Schoolmen will be glad to learn that the outlook at the present moment appears to include no threat of a paper shortage which will prevent the manufacture of all new textbooks required to meet the needs of this country for the next year.

Leon Henderson, Federal Price Administrator, in an address delivered at Hot Springs, Virginia, on November 13, 1941

before the Association of Advertisers and the American Association of Advertising Agencies said:

> According to present data the supplies of newsprint and book paper appear adequate for the next year in spite of the fact that defense activities are consuming about 20% of the nation's output. . . . Unfortunately, uninformed reports of a great paper shortage have tended to create a tight delivery situation on many kinds of paper and it is our information there exists rather extensive hoarding by some users. This condition has tended to magnify whatever shortage may exist and were it not for this fear it is our belief that supplies of paper at this time would be fairly adequate for practically all users.

Mr. Henderson's analysis of the situation is confirmed by the bulletin issued recently by the S. D. Warren Co., one of the largest manufacturers of paper in the United States. The bulletin states:

> The Government estimates that in 1942 it will require not more than 9 per cent of the capacity of book paper manufacturers. (The rest of the "20%" that Mr. Henderson cites apparently applies to newsprint.)
>
> The present capacity of the book paper industry has never been consumed in any one year.
>
> The orders for paper in 1940 represented only 77 per cent of the book paper capacity. 1940 was not a depression year. American business operated advantageously in 1940. There was no restriction on the consumption of paper in 1940, yet the demand represented only 77 per cent of the available productive capacity.
>
> If the Government will require only 9 per cent of the capacity of the book paper industry in 1942, the commercial users of book paper will be able to secure 91 per cent of capacity production, which is an increase of 14 per cent above 1940 orders for commercial use.

All this does not mean that no changes will occur in the textbook situation. Some changes already have been noted. There is a scarcity of bleaching materials which undoubtedly means that book papers will not be as white as they have been. Rising labor and material costs have resulted in price increases of many textbooks. Mr. J. R. Tiffany, general counsel of the Book Manufacturers Institute, states that within the past few years the cost of making a book has increased at least 35%; binder's board has increased 40% in the past year, cloth 25% and thread 30%, but actual increases in prices of books to consumers have not approached anything like these figures.

The wise school executive, even though he may feel assurance that he can get new books in 1942 to fill all his textbook needs will plan to have his textbook appropriations increased for the next twelve months to meet present and possible additional increases in prices. He will not delay too long in ordering what books he needs lest conditions not now predictable bring about a less favorable picture later in the year.

W. D. R.

IN OTHER PERIODICALS

By Nathan Lazar
The Bronx High School of Science, New York City

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3. Schaaf, William L., "The Mathematics Teacher's Bookshelf," pp. 14–15.
4. Mills, Leida H., and Stice, Gertrude Jackson, "Mary Kelly," p. 15.
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2. Peters, J. W., "The Theorem of Morley," pp. 119–126.
3. Gentry, F. C., "Analytic Geometry of the Triangle," pp. 127–140.
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5. Ballard, Ruth M., "Construction of Objective Tests in Mathematics," pp. 150–156.

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2. Hurlbert, Everett H., "A Simple Optical Device for Demonstrating the Conic Sections," pp. 828–831.
3. Stabler, E. R., "Another Viewpoint on Multiplication of Radicals," pp. 855–860.
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8. Richardson, R. G. D., "A New Mathematical Center," pp. 57–59.

Notice of Error!

On page 14 of the January (1942) issue of The Mathematics Teacher the Alabama representative of The National Council of Teachers of Mathematics should have been Mr. J. Eli Allen, not Mrs. Allen.—Editor.

The National Council of Teachers of Mathematics, Affiliated Clubs, 1941-1942*

CALIFORNIA

Mathematics Section of the California Teachers' Association, Bay Section.

Advisory Chairman: Miss Mary Banning, 1885 Melvin Rd., Oakland, Calif.

East Bay Chairman: Mr. J. H. Winegardner, 5727 Amy Drive, Oakland, Calif.

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West Bay Chairman: Mrs. Maud Oakes Volandri, Balboa High School, San Francisco, Calif.

West Bay Secretary-treasurer: Miss Norma Klans, Balboa High School, San Francisco, Calif.

Mathematics Teachers' Association of the North San Joaquin District.

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Secretary-treasurer: Mr. Ernest Eilertsen, 215 Olive Ave., Modesto, Calif.

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Secretary: Miss Ethel Spearman, P.O. Box 315, Sanger, Calif.

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Vice-President: Dr. A. E. Mallory, Colorado State College of Education, Greeley, Colo.

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Treasurer: Miss Ruth Layne, 1222 S. Emporia, Wichita, Kans.

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KENTUCKY

Kentucky Council of Mathematics Teachers.

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Secretary-treasurer: Miss Edith Wood, Okolona High School, Route 4, Louisville, Ky.

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MARYLAND

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* These clubs have reported to The National Council of Teachers of Mathematics. If your club is not listed among them, kindly ask your secretary to get in touch with Mr. Kenneth Brown, 525 W. 120th St., New York, N. Y.

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MISSISSIPPI (See Louisiana)

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